ECE 558

Project 1 Solutions

① Let A be the event that Inspector 1 finds a defective item

B be the — " ——— 2 — " ———

$Pr(A) = 0.8$, $Pr(B) = 0.7$

The probability of a defective item getting through is the probability that Inspector 1 does not find it and Inspector 2 does not find it.

Note: $\bar{A}$ is the event that Inspector 1 lets a defective item through

$\bar{B}$ — " ——— 2 — " ———

The Inspectors work alone, so the events A, B are independent.

$$\begin{aligned}
\therefore \; Pr(\bar{A} \cap \bar{B}) &= Pr(\bar{A}) \cdot Pr(\bar{B}) \\
&= [1 - Pr(A)] \cdot [1 - Pr(B)] \\
&= 0.2 \times 0.3 \\
&= 0.06
\end{aligned}$$

② Successes $= \{ (5,x) \cup (6,x) \cup (x,5) \cup (x,6) \}$ where x: don't care

Could enumerate successes; should find 20 successes.

Another approach is to let A be the event that die 1 shows 5 or 6

and let B — " ——— 2 — " —

Probability "die 1 shows 5 or 6" or "die 2 shows 5 or 6" is the union of the two events.

$Pr(A) = \frac{2}{6}$, $Pr(B) = \frac{2}{6}$, $Pr(A \cap B) = Pr(A) \cdot Pr(B)$ since indpt.

$$\begin{aligned}
Pr(A \cup B) &= Pr(A) + Pr(B) - Pr(A \cap B) \\
&= \frac{1}{3} + \frac{1}{3} - \frac{1}{9} \\
&= \frac{5}{9}
\end{aligned}$$

③ Three persons work indpt'ly on a design problem.

Let A be the event that person 1 solves the problem.

B — " ——— 2 — " ———

C — " ——— 3 — " ———

$Pr(A) = \frac{1}{4}$, $Pr(B) = \frac{1}{3}$, $Pr(C) = \frac{1}{2}$, $Pr(A \cap B \cap C) = Pr(A)Pr(B)Pr(C)$
since indpt.

Probability that problem is solved is the probability that #1 gets the solution, <u>or</u> #2 gets the solution, <u>or</u> #3 gets the solution.

$$Pr(A \cup B \cup C) = Pr(A) + Pr(B) + Pr(C) - Pr(A \cap B) - Pr(A \cap C) - Pr(B \cap C) + Pr(A \cap B \cap C)$$

$$= \frac{1}{4} + \frac{1}{3} + \frac{1}{2} - \frac{1}{12} - \frac{1}{8} - \frac{1}{6} + \frac{1}{24}$$

$$= \frac{3}{4}$$

Two ways to show:

a) consider that problem remains <u>unsolved</u> if all 3 persons fail

$$Pr(\text{solved}) = 1 - Pr(\text{unsolved}) = 1 - Pr(\bar{A} \cap \bar{B} \cap \bar{C})$$
$$= 1 - [Pr(\bar{A}) \cdot Pr(\bar{B}) \cdot Pr(\bar{C})]$$

b) Let D be the event that #1 gets solution <u>or</u> #2 gets sol'n.

$$Pr(D) = Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A) \cdot Pr(B)$$

Now, probability that problem is solved is the probability of C <u>or</u> D.

$$Pr(C \cup D) = Pr(C) + Pr(D) - Pr(C) \cdot Pr(D)$$
$$= Pr(C) + Pr(A) + Pr(B) - Pr(A)Pr(B) - Pr(C) \cdot Pr(D)$$
$$= Pr(A) + Pr(B) + Pr(C) - Pr(A)Pr(B) - [Pr(C)Pr(A) + Pr(C)Pr(B) - Pr(C)Pr(A)Pr(B)]$$
$$= Pr(A) + Pr(B) + Pr(C) - Pr(A)Pr(B) - Pr(A)Pr(C) - Pr(B)Pr(C) + Pr(A)Pr(B)Pr(C)$$

(4) Machines are indpt.

A: during any one hour, machine 1 requires no attention

B: ——— " ——— 2 ——— " ———

C: ——— " ——— 3 ——— " ———

$Pr(A) = 0.9$, $Pr(B) = 0.8$, $Pr(C) = 0.85$

a) probability that none requires attention is the probability that machine 1 requires no attention <u>and</u> machine 2 requires no attention, <u>and</u> machine 3 requires no attention.

$$Pr(A \cap B \cap C) = Pr(A) \cdot Pr(B) \cdot Pr(C) = 0.9 \times 0.8 \times 0.85 = 0.612$$

b) probability that at least one requires no attention is the probability that machine 1 requires no attention <u>or</u> machine 2 requires no attention <u>or</u> machine 3 requires no attention.

Note: this is the same as $1 - Pr$ (all 3 machines require attention)

$$Pr(A \cup B \cup C) = 1 - Pr(\bar{A} \cap \bar{B} \cap \bar{C}) = 1 - Pr(\bar{A})Pr(\bar{B})Pr(\bar{C})$$

$$= 1 - (1 - Pr(A))(1 - Pr(B))(1 - Pr(C)) = 1 - 0.1 \times 0.2 \times 0.15$$

$$= 0.997$$

(5) Six identical items. Three are defective.

a) Test the items one after the other, until the 3 defective items are found. Note: tests are <u>not</u> independent. Use conditional probability.

i) Pr(item 1 defective, then item 2 defective, then item 3 defective)

$$= \frac{3}{6} \times \frac{2}{5} \times \frac{1}{4} = \frac{1}{20}$$

Why? In first test there are 3 good items, and 3 bad items GGGBBB, so you have a $3/6$ chance of selecting a bad item. Then, in test #2, there is the following GGGBB, since one bad item has been removed. Test #3 has GGGB left, so you have a $1/4$ chance of choosing the last bad item.

ii) Pr (4 tests are required to find 3 bad items) =

Pr (1st item good, next 3 bad) + Pr (2nd item good, others bad) +

Pr (3rd item good, others bad)

Let G_i represent "good item drawn for ith test"

B_i —"— bad —"————

Pr (stop on 4th test) =

$$Pr(G_1) \cdot Pr(B_2 | G_1) \cdot Pr(B_3 | B_2 \cap G_1) \cdot Pr(B_4 | B_3 \cap B_2 \cap G_1)$$

$$+ Pr(B_1) \cdot Pr(G_2 | B_1) \cdot Pr(B_3 | G_2 \cap B_1) \cdot Pr(B_4 | B_3 \cap G_2 \cap B_1)$$

$$+ Pr(B_1) \cdot Pr(B_2 | B_1) \cdot Pr(G_3 | B_2 \cap B_1) \cdot Pr(B_4 / G_3 \cap B_2 \cap B_1)$$

$$= \frac{3}{6} \times \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} + \frac{3}{6} \times \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} + \frac{3}{6} \times \frac{2}{5} \times \frac{3}{4} \times \frac{1}{3} = \frac{3}{20}$$

b) $$Pr(G_1 | \text{stopped on } 4^{th} \text{ test}) = \frac{Pr(G_1 \cap \text{stopped on } 4^{th} \text{ test})}{Pr(\text{stopped on } 4^{th} \text{ test})} = \frac{1/20}{3/20} = \frac{1}{3}$$