Name: _________________________________

This is an open-book, open-notes exam. The use of electronic calculators is permitted. The exam lasts 75 minutes.

There are four questions on the exam. The first asks two “short answer” questions. The remaining three are problems to work. Do all your work on these pages and indicate your final answer clearly. For each of the problems I’ve provided an extra worksheet. Use the backs of the sheets if necessary. Neatness and clarity are important and can influence your grade!

Each problem is weighted toward the final total as shown below.

Grades

1. ___________________ (10 pts.)
2. ___________________ (30 pts.)
3. ___________________ (30 pts.)
4. ___________________ (30 pts.)
Total ________________ (100 pts.)
1. **[5 points each, total 10]** Please provide a short answer to part (a), and circle your choice to indicate your answer to (b).

(a) What is the difference between the classical approach and the Bayesian approach to detection?

(b) In a composite hypothesis test, the likelihood ratio is

\[
\Lambda(x) = \frac{p(x; \theta, H_1)}{p(x; \theta, H_0)}
\]

where \( \theta \) is a vector of unknown parameters. If for each \( \gamma \),

\[ Pr \{ \Lambda(x) > \gamma \mid H_0 \} \]

does not depend on \( \theta \), which one of the following is true.

- The MAP detector is the same as the ML detector.
- There is a uniformly most powerful test.
- The deflection coefficient equals one.

(Circle the true statement to indicate your answer.)
2. **[30 points]** The PDFs of a single observation $x$ under $\mathcal{H}_0$ and $\mathcal{H}_1$ are shown below.

Find $P_D$ as a function of $P_{FA}$ for the optimal Neyman-Pearson detector and sketch the ROC.
EXTRA WORKSHEET for problem 2
3. **[30 points]** We make two independent observations $x(0)$ and $x(1)$ with

$$x(n) \sim \mathcal{N}(0, \sigma^2) \text{ for } n = 0, 1 \text{ under } \mathcal{H}_0$$

and

$$x(0) \sim \mathcal{N}(\mu_{10}, \sigma^2) \quad x(1) \sim \mathcal{N}(\mu_{11}, \sigma^2) \quad \text{under } \mathcal{H}_1.$$ 

The means $\mu_{10}$ and $\mu_{11}$ are known, and we know from prior information that $\mathcal{H}_0$ is twice as likely to occur as $\mathcal{H}_1$.

Show that the minimum $P_e$ detector has the form

$$b^T \begin{bmatrix} x(0) \\ x(1) \end{bmatrix} \begin{cases} > d_1 & \text{if } H_0 \\ < d_0 & \text{if } H_1 \end{cases}$$

and find the values of $b$ and $c$. 
EXTRA WORKSHEET for problem 3
4. **[30 points]** You measure \( x(n) = \alpha n + w(n) \), \( n = 0, 1, \ldots, N - 1 \) where \( w(n) \) is WGN of known variance \( \sigma^2 \) but \( \alpha \) is unknown. Show that the GLRT \( L_G(x) \) for

\[
\mathcal{H}_0 : \alpha = 0 \\
\mathcal{H}_1 : \alpha \neq 0
\]

has the form

\[
\hat{\alpha}^2 \begin{cases} 
\gtrsim & \frac{d_1}{d_0} \frac{2 \sigma^2 \ln(\gamma)}{\sum_{n=0}^{N-1} n^2} \\
\lesssim & \frac{d_1}{d_0}
\end{cases}
\]

where \( \hat{\alpha} \) is the ML estimate of \( \alpha \). (Note: you will need to determine \( \hat{\alpha} \) to obtain this form of the GLRT.)
EXTRA WORKSHEET for problem 4