Name:  ____________________________________________

Instructions:
The examination lasts for two hours and is OPEN book, OPEN notes.

Do all your work on the pages in this exam booklet. Show your work for each problem on the same page as the problem and on the worksheet that follows each problem page. There is also an extra worksheet at the back, and you can also write on the backs of the pages if you need to. **Do not unstaple these pages. Any unstapled or restapled pages will NOT be graded.**

Show your work and clearly indicate your final answers. Neatness and organization in your work is important and will influence your grade.

Each problem is weighted toward the final total as shown below.

**Grades**

1. ____________________ (25 pts.)
2. ____________________ (25 pts.)
3. ____________________ (25 pts.)
4. ____________________ (25 pts.)

Total _____________ (100 pts.)

On the next pages are some formulas and diagrams that you may find useful.
For the step response of the continuous-time system

\[ G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \]

the percent overshoot \( M_p \), the rise time \( t_r \) and the settling time \( t_s \) are approximated by

\[ M_p \approx e^{-\pi\zeta/\sqrt{1-\zeta^2}} \quad 0 \leq \zeta < 1 \]
\[ t_r \approx \frac{1.8}{\omega_n} \]
\[ t_s \approx \frac{4.6}{\zeta\omega_n}. \]

(Note that \( \zeta = 0.5 \) for \( M_p = 0.16 \), and \( \zeta = 0.7 \) for \( M_p = 0.05 \).)

### DISCRETE EQUIVALENTS

<table>
<thead>
<tr>
<th>Forward rule</th>
<th>Backward rule</th>
<th>Bilinear transformation</th>
<th>ZOH({H(s)})</th>
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</thead>
<tbody>
<tr>
<td>( s \leftarrow \frac{z - 1}{T} )</td>
<td>( s \leftarrow \frac{z - 1}{Tz} )</td>
<td>( s \leftarrow \frac{2}{T} \frac{z - 1}{z + 1} )</td>
<td>( (1 - z^{-1})Z \left{ \frac{H(s)}{s} \right} )</td>
</tr>
</tbody>
</table>
1. [25 points] For the system

\[
\frac{1}{6} \frac{1}{z-1} \rightarrow u(k) \rightarrow \text{ZOH} \rightarrow \frac{\ln(3)}{s + \ln(3)} \rightarrow y(t)
\]

\[
r(k) = \begin{cases} 
  k + 1, & k \geq 0 \\
  0, & k < 0
\end{cases}
\]

Determine \( y(t) \) for \( t = 3 \) when \( r(k) \).

[BONUS (10 points): What is \( y(t) \) for \( t = \frac{3}{2} \), assuming the same \( r(k) \)?]
2. [25 points] A controller that combines state feedback with a predictor estimator is diagrammed below.

![Diagram of control system]

Equations describing System 1 (the plant) are

\[
x(k + 1) = \begin{bmatrix} \frac{1}{2} & \frac{-1}{4} \\ 0 & \frac{1}{4} \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k) \\
y(k) = \begin{bmatrix} 1 & 2 \end{bmatrix} x(k)
\]

and the equation describing System 2 (the estimator) is

\[
\hat{x}(k + 1) = \begin{bmatrix} \frac{1}{2} & \frac{-1}{4} \\ 0 & \frac{1}{4} \end{bmatrix} \hat{x}(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k) + \begin{bmatrix} \frac{-1}{8} \\ 0 \end{bmatrix} (y(k) - \begin{bmatrix} 1 & 2 \end{bmatrix} \hat{x}(k)) .
\]

If \( x(0) = [2 \ 2]^T \) and \( \hat{x}(0) = [0 \ 0]^T \), find

\[
y(k) - \begin{bmatrix} 1 & 2 \end{bmatrix} \hat{x}(k)
\]

for \( k = 2 \) in the following two cases.

(a) When \( r(k) = (1/2)^k + 3 (1/4)^k + 5 (1/8)^k + \sin(0.25k + \pi/16) \).

(b) When \( r(k) \) is a unit step.
WORKSHEET FOR PROBLEM 2
3. [25 points] Consider the following discrete-time system.

\[ u(k) \xrightarrow{\begin{array}{c} 0.5 \\ z - 0.5 \end{array}} y(k) \]

(a) Write down a state variable description for this system with \( x(k) = y(k) \).

(b) Develop of state-variable-based integral control that achieves a closed-loop transfer function

\[ \frac{Y(z)}{R(z)} = \frac{0.75}{z - 0.25} \]

If there are any pole/zero cancellations in \( Y(z)/R(z) \), force them to occur at \( z = 0 \).
4. [25 points] For the system

\[
x(k + 1) = \begin{bmatrix}
2.5 & -2.5 & 1.25 & -0.25 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix} x(k) + \begin{bmatrix}
1 \\
0 \\
0 \\
0
\end{bmatrix} u(k)
\]

\[y(k) = \begin{bmatrix}
0 & 1 & 4 & 3
\end{bmatrix} x(k)\]

and control law

\[u(k) = r(k) - K x(k),\]

determine a state feedback gain $K$ so that the closed loop system $Y(z)/R(z)$ settles in finite time to its final value in response to a step input $r(k)$. 
WORKSHEET FOR PROBLEM 4
EXTRA WORKSHEET (indicate problem number clearly)