Name: ____________________________________________

Instructions:
The examination lasts for 75 minutes and is closed book, closed notes.

Do all your work on the pages in this exam booklet. Show your work for each problem on the same page as the problem and on the worksheet that follows each problem page. Do not unstaple these pages. Any unstapled or restapled pages will NOT be graded.

Show your work and clearly indicate your final answers. Neatness and organization in your work is important and will influence your grade.

Each problem is weighted toward the final total as shown below.

Grades

1. ____________________ (20 pts.)
2. ____________________ (20 pts.)
3. ____________________ (20 pts.)
4. ____________________ (20 pts.)
5. ____________________ (20 pts.)

Total ________________ (100 pts.)

On the next pages are some formulas and diagrams that you may find useful.
\[ F(s) \quad f(kT) \quad F(z) \]
\[
\begin{array}{ccc}
\delta(k) & 1 & 1 \\
\delta(k - m) & z^{-m} & z^{-m} \\
\frac{1}{s} & 1(kT) & \frac{z}{z - 1} \\
\frac{1}{s^2} & kT & Tz \\
\frac{1}{s^2} & kT & \frac{(z - 1)^2}{z - 1} \\
\frac{1}{s + a} & \ e^{-akT} & \frac{z}{z - e^{-at}} \\
\frac{1}{(s + a)^2} & kT e^{-akT} & \frac{(z - e^{-at})^2}{z - 1} \\
\frac{a}{s(s + a)} & 1 - e^{-akT} & \frac{z(1 - e^{-at})}{(z - 1)(z - e^{-at})} \\
\end{array}
\]

For the step response of the continuous-time system

\[ G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \]

the percent overshoot \( M_p \), the rise time \( t_r \) and the settling time \( t_s \) are approximated by

\[ M_p \approx e^{-\pi\zeta/\sqrt{1 - \zeta^2}} \quad 0 \leq \zeta < 1 \]
\[ t_r \approx \frac{1.8}{\omega_n} \]
\[ t_s \approx \frac{4.6}{\zeta\omega_n} \]

(Note that \( \zeta = 0.5 \) for \( M_p = 0.16 \), and \( \zeta = 0.7 \) for \( M_p = 0.05 \).)

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DISCRETE EQUIVALENTS

- **Forward rule**
  \[ s \leftarrow \frac{z - 1}{T} \]

- **Backward rule**
  \[ s \leftarrow \frac{z - 1}{Tz} \]

- **Bilinear transformation**
  \[ s \leftarrow \frac{2z - 1}{Tz + 1} \]

\[ \text{ZOH}\{H(s)\} = (1 - z^{-1})Z \left\{ \frac{H(s)}{s} \right\} \]
1. [20 points] Given the basic feedback control system shown below, which pairs of transfer functions \( D(z) \) and \( G(z) \) guarantee a steady state error equal to zero for step reference inputs \( R(z) \)?

\[
\begin{align*}
R(z) & \quad E(z) \quad D(z) \quad G(z) \quad Y(z) \\
 & \downarrow \\
 & \quad - \\
 & \downarrow \\
 & \quad + \\
 & \quad - \\
\end{align*}
\]

(a) \( D(z) = \frac{0.5(z - 0.8)}{z} \), \( G(z) = \frac{z + 1}{(z - 1)(z - 0.8)} \)

(b) \( D(z) = \frac{0.5(z - 0.4)}{z - 1} \), \( G(z) = \frac{0.2}{z - 0.8} \)

(c) \( D(z) = 3 \), \( G(z) = \frac{z + 1}{(z - 1)^2} \)

(d) \( D(z) = \frac{3}{z} \), \( G(z) = \frac{0.2}{z - 0.8} \)
WORKSHEET FOR PROBLEM 1
2. **[20 points total]** You are considering discrete-time equivalents of a continuous-time transfer function

\[ D(s) = \frac{10s + 4}{s + 10}. \]

The bilinear transformation and the first-order-hold equivalent are discrete-time transfer functions of the form

\[ \hat{D}_bl(z) = \frac{a_1(z - b_1)}{z - c_1}, \quad \hat{D}_{lo}(z) = \frac{a_1(z - b_1)}{z - c_1}, \]

for sampling period \( T_1 \), and they are

\[ \hat{D}_bl(z) = \frac{a_2(z - b_2)}{z - c_2}, \quad \hat{D}_{lo}(z) = \frac{a_2(z - b_2)}{z - c_2} \]

for sampling period \( T_2 \).

(a) **[10 points]** If \( T_1 > T_2 \), do you expect

\[ |b_1 - \beta_1| < |b_2 - \beta_2| \quad \text{or} \quad |b_1 - \beta_1| > |b_2 - \beta_2|? \]

Justify your answer.

(b) **[10 points]** If \( T_1 > T_2 \), do you expect

\[ |\gamma_1 - 1| < |\gamma_2 - 1| \quad \text{or} \quad |\gamma_1 - 1| > |\gamma_2 - 1|? \]

Justify your answer.
3. **[20 points]** The closed loop system

The closed loop system

![System Diagram](image)

achieves a closed loop transfer function

\[
\frac{Y(z)}{R(z)} = \frac{0.825(z + 1)(z - 0.5)}{(z - 0.5 + j0.5)(z - 0.5 - j0.5)(z + 0.1)}
\]

when \(D(z)\) is

\[
D(z) = \frac{3(z - 0.5)}{z - 0.1}.
\]

This provides an acceptable step response, but the steady state error for a unit ramp input is too small by a factor of 10. Find a new choice for \(D(z)\) that meets the steady state error requirement while still maintaining an acceptable step response.
4. [20 points] Which of the two sampled-data control systems shown below has more overshoot in its step response? Justify your answer. (Please note that the sampling period $T$ is different in each diagram.)

\[
\begin{align*}
\text{System 1:} & & r(kT_1) & & + & & \frac{1}{2} & & u(kT_1) & & \frac{\ln(2)}{s + \ln(2)} & & y(t) \\
& & - & & \frac{1}{z - 1} & & \text{ZOH} & & T_1 = 1 \\
\text{System 2:} & & r(kT_2) & & + & & \frac{1}{3} & & u(kT_2) & & \frac{\ln(2)}{s + \ln(2)} & & y(t) \\
& & - & & \frac{1}{z - 1} & & \text{ZOH} & & T_2 = 2
\end{align*}
\]
5. [20 points] The Bode plot of a discrete-time transfer function $G(z)$ appears below.

![Bode plot](image)

Find $D(z)$ so that the closed loop system

![Closed loop system diagram]

has a phase margin of $50^\circ$. 
WORKSHEET FOR PROBLEM 5