Name: __________________________________________

Student #: ______________________________________

Instructions:
The examination lasts for 120 minutes. You may bring one sheet (8 1/2 by 11 inches), both
sides, of notes, but otherwise the exam is closed book.
Six examination questions are given on a separate set of pages. You may keep the questions
after the exam.
Do all your work on the pages attached to this sheet. Do not unstaple these pages. Any
unstapled or restapled pages will NOT be graded.
Show your work and clearly indicate your final answers. Neatness and organization in your
work is important and will influence your grade.
Each problem is weighted toward the final total as shown below.

Grades

1. __________________ (15 pts.)
2. __________________ (15 pts.)
3. __________________ (15 pts.)
4. __________________ (20 pts.)
5. __________________ (20 pts.)
6. __________________ (15 pts.)
Total _______________ (100 pts.)

On the next page are some formulas that you may find useful.
For the step response of the continuous-time system
\[
G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2},
\]
the percent overshoot \( M_p \), the rise time \( t_r \) and the settling time \( t_s \) are approximated by
\[
M_p \approx e^{-\pi\zeta/\sqrt{1-\zeta^2}} \quad 0 \leq \zeta < 1
\]
\[
t_r \approx \frac{1.8}{\omega_n}
\]
\[
t_s \approx \frac{4.6}{\zeta\omega_n}
\]

DISCRETE EQUIVALENTS

<table>
<thead>
<tr>
<th>Forward rule</th>
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</thead>
<tbody>
<tr>
<td>( s \leftarrow \frac{z-1}{T} )</td>
<td>( s \leftarrow \frac{z-1}{Tz} )</td>
<td>( s \leftarrow \frac{2(z-1)}{T(z+1)} )</td>
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\[
\text{ZOH}\{H(s)\} = (1 - z^{-1}) \mathcal{Z} \left\{ \frac{H(s)}{s} \right\}
\]
1. A continuous-time compensator \( D(s) \) has transfer function

\[
G(s) = \frac{3(s + 0.01)(s + 5)}{(s + 10)(s + 0.001)}.
\]

If \( \hat{G}(z) \) is the discrete equivalent of \( G(s) \) obtained by applying the prewarped bilinear transformation at a sampling period of \( T = 0.01 \) sec, prewarped to 10 rad/sec, then

(a) what is the value of \( \hat{G}(1) \)?
(b) what is the value of \( \hat{G}(-1) \)?
(c) what is the value of \( \hat{G}(e^{j0.1}) \)?

2. The following questions pertain to the system shown below.

(a) Find an expression in terms of \( K \) and \( H(s) \) for the steady state error

\[
\lim_{k \to \infty} e(k) = r(k) - y(k)
\]

when \( r(k) \) is a unit step, assuming the closed loop is stable.

(b) Give an argument to support or refute the following statement.

Assuming that \( H(s) \) has two stable, real poles and no finite valued zeros, then as \( K \) is increased toward infinity, the closed loop system eventually becomes unstable.

3. Design \( D(z) \) in

\[
\begin{array}{c}
r(k) \\
+ \\
- \\
\hline
D(z) \\
0.01(z + 0.9) \\
(z - 0.9)(z - 0.8)
\end{array}
\]

so that when \( r(k) \) is a unit step, \( y(k) = 1 \) for all \( k \geq 2 \).
4. For the combined plant/controller/estimator given by
   \[ x(k+1) = Ax(k) + Bu(k) \]
   \[ y(k) = Cx(k) \]
   \[ u(k) = r(k) - K\hat{x}(k) \]
   \[ \hat{x}(k+1) = A\hat{x}(k) + Bu(k) + L[y(k) - C\hat{x}(k)] \]
   (a) write down a state space description for a system with input \( r(k) \) and output \( u(k) \)
   and with a state vector given by \( [x(k) \; \hat{x}(k)]^T \), where \( \hat{x}(k) = x(k) - \hat{x}(k) \);
   (b) find an expression for the transfer function \( U(z)/R(z) \).

5. (a) Verify that
   \[ x(k+1) = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} T^2/2 \\ T \end{bmatrix} u(k) \]
   \[ y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(k) \]
   is one possible state space description for the ZOH-equivalent of \( G(s) = 1/s^2 \) with
   sampling period \( T \).
   (b) Find \( T \) so that when \( \xi(k) = Tx(k) \),
   \[ \xi(k+1) = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix} \xi(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k) \]
   \[ y(k) = \begin{bmatrix} T^2/2 & 0 \end{bmatrix} \xi(k) \]
   (i.e. so that the system is in controllable canonical form).
   (c) Find \( K_\xi \) so that
   \( u(k) = r(k) - K_\xi \xi(k) \)
   places closed loop poles at the roots of \( z^2 - 1.6z + 0.7 \).
   (d) Find \( K_x \) so that
   \( u(k) = r(k) - K_x x(k) \)
   places closed loop poles at the roots of \( z^2 - 1.6z + 0.7 \).

6. A predictor estimator with estimator gain matrix \( L_p = [0 \; 2]^T \) places both of the
   estimator poles of
   \[ x(k+1) = \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u(k) \]
   \[ y(k) = \begin{bmatrix} 1 & 1 \end{bmatrix} x(k) \]
   at \( z = 0 \). What value of current estimator gain \( L_c \) will do the same?