Name: ________________________________________________

Instructions:
The examination lasts for 75 minutes and is closed book, closed notes.
Do all your work on the pages in this exam booklet. Do not unstaple these pages. Any unstapled or restapled pages will NOT be graded.
Show your work and clearly indicate your final answers. Neatness and organization in your work is important and will influence your grade.

Each problem is weighted toward the final total as shown below.

Grades

1. __________________ (30 pts.)
2. __________________ (10 pts.)
3. __________________ (10 pts.)
4. __________________ (20 pts.)
5. __________________ (30 pts.)
Total __________________ (100 pts.)

On the next page are some formulas that you may find useful.
For the step response of the continuous-time system
\[ G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \]
the percent overshoot \( M_p \), the rise time \( t_r \) and the settling time \( t_s \) are approximated by
\[
M_p \approx e^{-\pi\zeta/\sqrt{1-\zeta^2}} \quad 0 \leq \zeta < 1 \\
t_r \approx \frac{1.8}{\omega_n} \\
t_s \approx \frac{4.6}{\zeta\omega_n}.
\]

**DISCRETE EQUIVALENTS**

<table>
<thead>
<tr>
<th>Forward rule</th>
<th>( s \leftarrow \frac{z - 1}{T} )</th>
</tr>
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<tbody>
<tr>
<td>Backward rule</td>
<td>( s \leftarrow \frac{z - 1}{Tz} )</td>
</tr>
<tr>
<td>Trapezoid rule</td>
<td>( s \leftarrow \frac{2(z - 1)}{T(z + 1)} )</td>
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\[ \text{ZOH}\{H(s)\} = (1 - z^{-1})Z \left\{ \frac{H(s)}{s} \right\} \]
1. [6 points each, total 30] Answer true or false.

(a) The ZOH-equivalent of \( G(s)/(1 + G(s)) \) describes the behavior at sampling instants of the closed loop system shown below.

(b) As \( T \to 0 \) the poles of the ZOH-equivalent of \( G(s) \) approach \( z = 1 \).

(c) In a stable digital lead compensator the zero is located on the real axis in the \( z \)-plane nearer to \( z = -1 \) than the pole.

(d) If \( G(s) \) has a pole at \( s = s_0 \), then the bilinear transformation of \( G(s) \) has a pole at \( z = e^{s_0 T} \).

(e) If \( G(s) \) has a pole at \( s = s_0 \), then the ZOH-equivalent of \( G(s) \) has a pole at \( z = e^{s_0 T} \).
2. [10 points] The figures below show the magnitude responses of three discrete-time linear systems. One of them is the magnitude response of the ZOH-equivalent of

$$G(s) = \frac{4(s + 1)}{(s + 0.5)(s + 10)}$$

at some sampling period $T$, and another of the three is the magnitude response of the bilinear transformation of this $G(s)$ with the same $T$. Determine which of (a), (b) and (c) these are.
3. [10 points] Indicate the region of the z-plane corresponding to a complex pole pair $a$ and $a^*$ such that

$$G(s) = \frac{\alpha(z + 1)}{(z - a)(z - a^*)}$$

has overshoot less than 5% and $t_r < 1.8$ sec. Assume that $T = 0.6$ sec.
4. [10 points each, total 20] Consider the sampled data control system composed of the continuous-time plant with transfer function

\[
\frac{Y(s)}{U(s)} = \frac{6}{s + 3}
\]

and the feedback control law

\[u(t) = u(kT) = K [r(kT) - y(kT)] \text{ for } kT \leq t < (k + 1)T.\]

(a) For \(K = 1\) determine the maximum value of the sample period \(T\) for which the closed-loop system is stable.

(b) For \(T = 1/6\) seconds, determine the range of \(K\) (over both positive and negative values) for which the closed-loop system is stable.
5. **[5 points each, total 30]** Consider the discrete-time system described by

\[
\begin{align*}
x(k+1) &= \begin{bmatrix} 0.6 & -0.2 \\ 0.1 & 0.9 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(k) \\
y(k) &= \begin{bmatrix} 3 & -1 \end{bmatrix} x(k) + [1] u(k)
\end{align*}
\]

(a) Compute the transfer function.

(b) Specify the real and imaginary parts (to 2 decimal point accuracy) of all poles of the system.

(c) Specify the real and imaginary parts (to 2 decimal point accuracy) of all zeros of the system.

(d) Is this system stable? Justify your answer.

(e) Does the zero initial condition unit step response of this system take on both positive and negative values for \( k > 0 \)? Justify your answer.

(f) Is the largest steady-state sinusoidal response gain of this system greater than 25dB for any frequency sinusoidal input? Justify your answer.
[Extra worksheet. Indicate the problem number clearly.]
[Extra worksheet. Indicate the problem number clearly.]