Name: __________________________________________________________

Student #: ______________________________________________________

Instructions:
The examination lasts for 120 minutes and is closed book, closed notes.

Six examination questions are given on a separate set of pages. You may keep the questions after the exam.

Do all your work on the pages attached to this sheet. Do not unstaple these pages. Any unstapled or restapled pages will NOT be graded.

Show your work and clearly indicate your final answers. Neatness and organization in your work is important and will influence your grade.

Each problem is weighted toward the final total as shown below.

Grades

1. ______________________ (15 pts.)
2. ______________________ (15 pts.)
3. ______________________ (15 pts.)
4. ______________________ (20 pts.)
5. ______________________ (20 pts.)
6. ______________________ (15 pts.)

Total __________________ (100 pts.)
1. You are given the system
\[
x(k + 1) = \begin{bmatrix} 3 & 4 \\ -1 & -1 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k) \\
y(k) = \begin{bmatrix} 1 & 2 \end{bmatrix} x(k)
\]
(a) Find the transfer function \( Y(z)/U(z) \).
(b) Is the system controllable? Support your answer.
(c) Is the system observable? Support your answer.
(d) Sketch the root locus for the closed loop system obtained with
\[
u(k) = K [r(k) - y(k)]
\]
as \( K \) varies between 0 and \( \infty \).

2. For the system
\[
x(k + 1) = \begin{bmatrix} 2.5 & -2.5 & 1.25 & -0.25 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} u(k) \\
y(k) = \begin{bmatrix} 0 & 1 & 4 & 3 \end{bmatrix} x(k)
\]
and control law
\[
u(k) = r(k) - K x(k)
\]
determine a state feedback gain \( K \) so that the closed loop system \( Y(z)/R(z) \) settles in finite time to its final value in response to a step input \( r(k) \).

3. Write down three different state space realizations for the discrete-time system with transfer function
\[
\frac{Y(z)}{U(z)} = \frac{1}{(z - 1)(z - \frac{1}{2})(z - \frac{1}{4})}
\]
4. The following questions pertain to the system shown below.

![System Diagram]

(a) Find an expression in terms of $K$ and $H(s)$ for the steady state error
\[ \lim_{k \to \infty} e(k) = r(k) - y(k) \]
when $r(k)$ is a unit step, assuming the closed loop is stable.

(b) Give an argument to support or refute the following statement.
Assuming that $H(s)$ has two stable, real poles and no finite valued zeros, then as $K$ is increased toward infinity, the closed loop system eventually becomes unstable.

5. Let $x_c(k)$ be the state vector for the controllable canonical realization for
\[ H(z) = \frac{Y(z)}{U(z)} = \frac{z^2 + 3z + 2}{z^3 - 1.5z^2 + z - 0.25}, \]
and let $x_o(k)$ be the state vector for the observable canonical realization for the same system. For this system, assume that
\[ x_c = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} x_o. \]

(a) Find $K_c$ for a state feedback law
\[ u(k) = -K_c x_c(k) \]
that places all poles of the controlled system at the roots of
\[ z^3 - 0.5z^2 - 0.5z + 0.5. \]

(b) Find $L_c$ for a predictor estimator
\[ \hat{x}_c(k+1) = A_c \hat{x}_c(k) + B_c u(k) + L_c [y(k) - C_c \hat{x}_c(k)], \]
where $A_c$, $B_c$, $C_c$ are the matrices for the controllable canonical state description, so that the estimator poles are at the roots of
\[ z^3 - 0.5z^2 - 0.5z + 0.5. \]

6. Let $H(s) = H_1(s)H_2(s)$.

(a) If $\tilde{H}_1(z)$ and $\tilde{H}_2(z)$ are the bilinear transformations of $H_1(s)$ and $H_2(s)$, respectively, is $\tilde{H}_1(z)\tilde{H}_2(z)$ the bilinear transformation of $H(s)$? Justify your answer.

(b) Now let $\tilde{H}_1(z)$ and $\tilde{H}_2(z)$ be the corresponding ZOH-equivalents. Is $\tilde{H}_1(z)\tilde{H}_2(z)$ the ZOH-equivalent of $H(s)$? Justify your answer.