Cognitive Radio Based State Estimation in Cyber-Physical Systems

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Abstract—We investigate the state estimation problem in cyberphysical systems (CPS) where the dynamical physical process is measured by a wireless sensor and the measurements are transmitted to a remote state estimator. It has been shown that the estimation performance strongly depends on the wireless communication quality. To enhance the estimation performance, we apply the cognitive radio technique to the system and propose a CHAnnel seNsing and switChing meChanism (CHANCE) to explore opportunistic accessibility of multiple channels. We consider two types of wireless channels, i.e., one unlicensed channel which can be accessed freely and several licensed channels which have been pre-assigned to primary users. For the single-licensed-channel case, we develop a necessary condition for the estimation stability based on the physical process dynamics, channel quality and the channel sensing accuracy. This condition becomes also sufficient under certain conditions. We also derive the conditions under which the estimation performance is guaranteed to be improved by CHANCE. The above results are then extended to multi-licensed-channel cases. Simulations based on a particular linear system show that, the long-run mean estimation error covariance with CHANCE is at least 63% less than that without CHANCE. It is also shown that CHANCE outperforms the existing RANDOM mechanism in terms of estimation performance.

Index Terms—Cognitive radio; cyber physical system; state estimation; stability condition; performance bounds

I. INTRODUCTION

The emerging cognitive radio (CR) technology which allows dynamic spectrum reuses can significantly improve spectrum utilization efficiency and mitigate the situation of spectrum scarcity [1], [2]. With CR, unlicensed users can opportunistically use holes of the licensed spectrum, by which the communication quality of the unlicensed users is expected to be improved. A considerable amount of literature has been published that studies various aspects of the CR networks such as spectrum sensing algorithms, network protocols, resource allocation, and network security [3]–[5].

In this paper, we study the CR technology from a cyber physical system (CPS) perspective. CPS systematically integrate physical processes with sensors, actuators and efficient computation and communication modules to provide real-time monitoring, control and other services. CPS facilitate intimate interactions between human beings and the physical world in a wide range of applications such as environment surveillance, vehicular networks, industrial automation and smart grid [6]–[10]. The close interplay between the cyber and physical spaces often requires real-time operation, efficient system resource utilization and security guarantee [7], [8].

In many CPS applications, sensor data often need to be transmitted through the wireless media to remote units for state estimation which plays a vital role in provisioning real-time monitoring and control. For example, in a smart grid, field sensors report real-time voltage and phase observations to remote power generators which estimate the real voltage and take certain actions in order to stabilize it within some desired range [6]. Due to a variety of factors such as bandwidth limitation, interferences from other wireless devices or environment, and channel fading, wireless communications usually undergo packet losses and delay which impose strict requirements on the communication protocols as well as estimation and control policies [11], [12]. Since the estimation mechanism relies on the information received from sensors, packet losses strongly affect the estimation performance. When a loss occurs, the estimator will have to “guess” current state of the physical process based on all historically received data, which will introduce error to the estimation performance. Illustrations of such effect are shown in Fig. 1 where we consider a simple i.i.d. packet loss model. The first figure shows that a packet loss will cause the estimation deviate from the one without loss, and the deviation is enlarged when packets are lost successively; the other figure demonstrates that the estimation performance degrades as the loss rate increases.

The problem of state estimation stability under lossy wireless communications has attracted intensive research attentions [13]–[16]. Considering the i.i.d packet loss model, Sinopoli et al. prove the existence of a critical value for the packet loss rate below which the estimation error covariance would converge in the mean square sense [13]. The estimation stability under a more complicated channel model with Markovian packet losses has been studied in [14]. A class of more general packet loss models based on semi-Markov chains is investigated in [15]. In order to preserve the estimation performance, sensor scheduling and transmission power control methods have been proposed to improve the communication quality [17], [18]. Basically, due to bandwidth limitation, a single channel may be insufficient for estimating fast changing physical processes. For state estimation with multiple sensors,
Chiuso et al. prove that the optimal estimation performance under packet loss cannot be reached using a single bandwidth-limited channel [19]. Liu et al. study the estimation stability over multiple wireless channels by dividing the measurement data and then transmit them through the channels separately [20]. However, they focus on fixed communication structures without considering channel dynamics.

We apply CR over multiple channels to enhance the state estimation performance in CPS. While it is well noticed that CR can effectively improve spectrum efficiency in the communication regime, its ability has not been fully exploited in the CPS domain. In the literature, only a few works have investigated the estimation and control problems with CR. In [21], Ma et al. propose an optimal estimation and control algorithm and analyze the system stability conditions focusing on a single licensed channel. The results have been extended to multi-channel cases [22]. However, in their work, only one licensed channel is sensed and opportunistically used for each transmission, which is different from ours where both licensed and unlicensed channels can be used. In addition, they do not consider channel sensing inaccuracy which is a fundamental problem for practical cognitive radio systems [23].

We propose a CR based CHANnel seNsing and switChing mEchanism (CHANCE) for state estimation for a class of generic CPS with linear state dynamics. We aim to answer whether and how the state estimation can be improved by the new mechanism. Different from classical CR theories which focus on network throughput, we emphasize on the communication reliability and state estimation performance. Our main contributions can be summarized as follows: (1) We propose a new state estimation framework by taking the advantage of the CR technology, and identify a novel packet loss model. (2) For the case that only one licensed channel is available, we derive a necessary condition for the estimation stability which takes factors from both physical and cyber spaces into account. This condition is also sufficient when the observation matrix has full column rank. Moreover, this condition is a generalization of existing results for either i.i.d. or Markov packet loss models. (3) We obtain the conditions under which the proposed mechanism can certainly improve the estimation performance. Moreover, a pair of upper and lower bounds for the mean square estimation error is determined. The performance improvement in terms of performance ratio is analyzed and the worst case ratio is derived. (4) We generalize the above results to accommodate multiple licensed channels. Corresponding stability conditions and performance bounds are also derived. Numeric results are provided to demonstrate the effectiveness of our proposed mechanism.

The remainder of this paper is organized as follows. The whole system is modeled in Section II. For the single-licensed-channel case, the state estimation stability conditions and performance are studied in Section III. Extensions to multi-licensed-channel cases are provided in Section IV. Section V presents simulations results. Finally, Section VI concludes the paper. To be concise, we present all main results in form of theorems to which the proofs can be found in the Appendices.

II. SYSTEM MODELING

We consider a class of generic CPS where the physical process has linear state dynamics. As shown in Fig. 2, a wireless sensor periodically samples the physical process and transmits its measurement to a remote estimator for state estimation. There is an unlicensed channel, $CH_0$, which is likely to be used by a large amount of wireless devices and thus packet collision may happen frequently. Alternatively, the sensor can opportunistically use the spectrum holes of several licensed channels (sub-bands divided from a wide-band channel), $\{CH_i|i=1,\ldots,m\}$, each of which is restricted to only a primary user (PU) that may not always occupy it. The basic idea of CHANCE is to apply CR to use spectrum holes of licensed channels for measurement transmission. Here, we assume all the channels are mutually independent [24], [25]. We adopt the “listen before talk” strategy such that the sensor performs channel sensing right before each data reporting time.

In the following, we present system models, where the main notations are listed in Table I. We adopt the following convention: $\mathbb{R}^n$ is the $n$-dimensional Euclidean space. $\mathbb{E}[^\cdot]$ and $\mathbb{P}[^\cdot]$ denote the expectation and probability of a random variable, respectively. We use boldface letters in lower case to denote vectors, and capital letters to denote matrices. For any matrix/vector $M$, $M'$ and $M^H$ denote the transpose and Hermitian transpose of $M$, respectively. $[^i]$ denotes the $i$-th entry of a vector, while $[^{i,j}]$ denotes the $(i,j)$-th entry of a matrix. $\text{Tr}(\cdot)$ and $\rho(\cdot)$ denote the trace and spectral radius of a square matrix, respectively. Denote $\mathbb{S}_+^n$ as the set of positive semi-definite matrices of dimension $n \times n$. $\forall X \in \mathbb{S}_+^n$, $\lambda_{\text{min}}(X)$ and $\lambda_{\text{max}}(X)$ are its minimum and maximum eigenvalues, respectively. $\forall X,Y \in \mathbb{S}_+^n$, $X \geq Y$ if $X - Y \in \mathbb{S}_+^n$.

A. The Physical Process and State Estimation Algorithm

We consider that the physical process runs with the following discrete-time state dynamics:

$$x_{k+1} = Ax_k + B a_k + w_k, \quad (1)$$

where $x_k \in \mathbb{R}^n$ is the state system, $a_k \in \mathbb{R}^m$ is the control action, $w_k \in \mathbb{R}^n$ is the system noise. As a typical and generic
The CHANCE mechanism, the packet loss process is neither i.i.d. nor Markovian as discussed in Remark 1. Therefore, we derive new necessary and sufficient conditions for the MSE stability of the state estimation.

B. CHANCE in the Cyber Space

The sensor is equipped with one antenna that can sense the signals in the licensed and unlicensed channels. To handle multi-channel sensing, sequential sensing (SS) and wideband sensing (WS) are two candidate methods [3]. With SS, the sensor scans the channels one-by-one until finding an idle channel and then transmits its data on it. In this way, the sensor can apply low-complexity spectrum sensing strategy, e.g., energy detection. In contrast, WS is able to handle multiple channels at the same time; however, it usually requires the sensor to be enough powerful for carrying out high-complexity calculations [3], [28]. In addition, since SS only sense a subset of all the channels in each round, it is likely to consume less energy than WS. Therefore, we apply SS in our study.

As shown in Fig. 2, CHANCE runs as follows. In each step, the sensor first scans the licensed channels following an ordered channel index set $Q_m$. It will stop sensing and transmit packet through the first channel that is sensed idle (unoccupied by its PU). If all the licensed channels in $Q_m$ are sensed busy, the sensor will transmit over the original channel $C_{H_0}$. During scanning, the channel sensing time spent on channel $C_{H_i}$ is $\tau_i$. Assume that $\tau_i$ is small enough such that the maximum scanning time will not exceed one step, i.e., $\sum_{i=1}^{m} \tau_i < T$, and that the channel state does not change during $\tau_i$. Without loss of generality, let $Q_m = \{1, \ldots, m\}$.

For each $i \in Q_m$, denote the state of $C_{H_i}$ at the time when the sensor senses $C_{H_i}$ in step $k$ as $s_{i,k} \in \{0, 1\}$. $s_{i,k} = 1$ if $C_{H_i}$ is busy and $s_{i,k} = 0$ otherwise. Denote $\omega_{i,k} \in \{0, 1\}$ as the sensing result such that $\omega_{i,k} = 0$ if $C_{H_i}$ is sensed idle; in this case, the sensor will switch to $C_{H_i}$ for transmission.

Conversely, $\omega_{i,k} = 1$ indicates that $C_{H_i}$ is sensed busy. Once a transmission on $C_{H_i}$ is completed, the sensor switches back to $C_{H_0}$ in order to minimize possible interference to the PU.

Channel Modeling: We assume that transmissions on $C_{H_0}$ suffer from i.i.d. packet losses with average loss rate $\ell_0$. For each licensed channel $C_{H_i}$, define its idle state loss rate $\ell_i$ as the loss rate of packets transmitted on $C_{H_i}$ when $C_{H_i}$ is idle, to account for factors such as path fading and environmental noise. The PU’s activities on $C_{H_i}$ are modeled as a continuous-time Markov process: the busy and idle periods of $C_{H_i}$ obey exponential distributions with means $1/\omega_{i,1}$ and $1/\omega_{i,0}$, respectively. In case $\omega_{i,1} = 0$ (or $\omega_{i,0} = 0$), $C_{H_i}$ becomes always busy (or idle). Under the hypothesis that $\omega_{i,1}$ and $\omega_{i,0}$ are known parameters, performing channel sensing on a constantly busy (idle) channel is unnecessary. Therefore, we assume $\omega_{i,1} = \omega_{i,0} > 0$ in the sequel. The following proposition derives the probability transition matrix $\Phi_i$ of $C_{H_i}$ where $[\Phi_i]_{jl} := P\{s_{i,k+1} = j | s_{i,k} = l\}$. Its proof can be found in Appendix A.

We assume that there is another common control channel for the sensor to inform the estimator to receive on $C_{H_i}$ [29].

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1Since $P_{0|0} > 0$, by the second line of (3), $P_k \in S_Q, \forall k > 0$. It is without loss of generality to assume $P_0 \in S_Q$.

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Table I

Notations | Description
---|---
$x_k, y_k$ | System state and measurement of the physical process
$C_{H_0}, C_{H_i}$ | The original channel and the $i$-th licensed channel, respectively. $i \in \{1, \ldots, m\}$
$\ell_0, \ell_i$ | Packet loss rate on $C_{H_0}$ and the loss rate on $C_{H_i}$ when $C_{H_i}$ is idle
$T, \tau$ | Sensor’s sampling period and channel sensing time
$s_{i,k}, \omega_{i,k}$ | The channel state and channel sensing result of $C_{H_i}$
$\alpha_i, \beta_i$ | $C_{H_i}$’s state transition probabilities. See (5)
pd,i, pf,i | Correct detection and false detection probabilities, respectively, for sensing $C_{H_i}$
Proposition 1: \( \{s_{i,k}\}_{k \geq 0} \) forms a homogeneous Markov chain with transition probability matrix
\[
\Phi_i = \begin{bmatrix}
1 - \alpha_i & \alpha_i \\
\beta_i & 1 - \beta_i
\end{bmatrix}
\]
where \( \alpha_i = \frac{\omega_{1,0}}{\omega_{1,0} + \omega_{0,0}} \left(1 - e^{-(\omega_{1,0} + \omega_{0,0})}T\right) \) and \( \beta_i = \frac{\omega_{1,0}}{\omega_{1,0} + \omega_{0,0}} \left(1 - e^{-(\omega_{1,0} + \omega_{0,0})}T\right) \).

Remark 1: Since \( \omega_{1,1}\omega_{0,0} > 0, 0 < \alpha_i + \beta_i < 1 \). For finite sampling period \( T \), the sequence \( \{s_{i,k}, \gamma_k\} : k \geq 0 \) forms a hidden Markov process with \( s_{i,k} \) as the hidden state and \( \gamma_k \) as the observation [30]. Due to imperfect channel sensing (see below), present packet loss rate may depend on all historical data. In other words, the packet loss process under CHANCE is generally neither an i.i.d. process nor a Markov chain, making many existing results in the literature inapplicable.

Remark 2: Above we describe a simple model for \( CH_1 \). However, the results obtained in this paper also hold for more general models, providing that \( \{s_{i,k}\}_{k \geq 0} \) is a homogeneous Markov chain with known transition probability matrix \( \Phi_i \) and that \( 0 < \alpha_i + \beta_i < 1 \). For example, the semi-Markov model presented in [22] can be adopted to allow any given homogeneous distributions of the busy/idle periods. In this case, the transition probability matrix \( \Phi \) can be obtained by the method proposed in [31]. We will compare the performance of CHANCE and the strategy proposed in [22] in Section V.

Channel Sensing: There are a few channel sensing techniques available, such as energy detection (which is simple but has relatively low accuracy) and feature detector (which is more accurate but requires prior knowledge of the PU’s signals) [32]. To characterize the channel sensing accuracy, we define two conditional probabilities, i.e., \( p_{d,i} = \mathbb{P}\{\alpha_i = 0|s_{i,k} = 0\} \) and \( p_{f,i} = \mathbb{P}\{\alpha_i = 0|s_{i,k} = 1\} \), as the correct detection and false detection probabilities, respectively. For perfect channel sensing, \( p_{d,i} = 1 \) and \( p_{f,i} = 0 \). Generally, \( p_{d,i} \) and \( p_{f,i} \) highly depend on the sensing technique. It is worth noticing that our results do not rely on any particular formulas of the above two probabilities.

III. THE SINGLE-LICENSED CHANNEL CASE

We begin with the simple single-licensed-channel case (i.e., \( m = 1 \)) while extensions to general cases are presented in the next section. In this section, we first study the conditions for the estimation stability. Then we show how CHANCE can expand the estimation stability region. We analyze the estimation performance bounds and find the conditions when CHANCE certainly improves the performance.

Suppose the licensed channel is \( CH_1 \). The suffix 1 indicating \( CH_1 \) in the above defined variables is dropped in this section since the indication is clear. Denote the conditional packet loss probabilities on the state of \( CH_1 \) by \( \psi_1 = \mathbb{P}\{\gamma_k = 0|s_{k-1} = 0\} \) and \( \psi_2 = \mathbb{P}\{\gamma_k = 0|s_{k-1} = 1\} \).

In case that \( CH_1 \) is idle, the measurement packet will get lost with probability equals to either \( \ell_1 \) (if the channel sensing result is \( \alpha_k = 0 \) and thus the packet is delivered over \( CH_1 \)) or \( \ell_0 \) (if \( \alpha_k = 1 \) and thus packet is transmitted on \( CH_0 \)). Therefore, \( \psi_1 \) can be expressed below. The expression for \( \psi_2 \) is similar.

\[
\begin{align*}
\psi_1 &= p_d\ell_1 + (1-p_d)\ell_0 \triangleq \psi_1(p_d), \\
\psi_2 &= p_f + (1-p_f)\ell_0 \triangleq \psi_2(p_f)
\end{align*}
\]  

Note that the two equations in (6) are asymmetric because a packet will get lost at rate \( \ell_1 \) under correct detection, but will certainly lose under false detection. Define the sensing matrix \( \Psi \triangleq \text{Diag}(\psi_1, \psi_2) \) and the critical matrix \( \Phi \Psi = \left[\begin{array}{cc} \psi_1(1-\alpha) & \psi_2\alpha \\
\psi_1\beta & \psi_2(1-\beta) \end{array}\right] \).

A. Stability Conditions

Theorem 1: A necessary condition for the MSE stability of the estimation applied with CHANCE is:

\[
\sigma_d \rho(A)^2 < 1. \tag{7}
\]

Theorem 2: If \( C \) has full column rank, condition (7) is both necessary and sufficient.

Remark 3: The above result is more general than some existing conditions. For example,

1) in the special case with \( p_d = p_f = 0, CH_1 \) will never be accessed, i.e., all the transmissions are carried out over \( CH_0 \) with i.i.d. packet loss rate \( \ell_0 \). In this case, \( \psi_1 = \psi_2 = \ell_0, \sigma_2 = \ell_0 \), and condition (7) reduces to \( \ell_0 \rho(A)^2 < 1 \) which reduces existing result in [13].

2) in the special case with \( \ell_1 = 0 \) and \( p_f = p_d = 1 \), all the transmissions are carried out over \( CH_1 \) and the packet loss process obeys Markov distribution with recovery rate \( \mathbb{P}\{\gamma_{k-1} = 1|\gamma_k = 0\} = \mathbb{P}\{s_{k} = 0|s_{k-1} = 1\} = \beta \).

In this case, \( \psi_1 = 0, \psi_2 = 1 \) and \( \sigma_2 = 1 - \beta \). As a result, condition (7) reduces to \( (1-\beta)\rho(A)^2 < 1 \) which reduces the recovery condition proposed in [14].

Remark 4: Condition (7) provides a joint condition for both physical and cyber spaces. It can be interpreted as follows. If we define \( \tilde{s}_k = (\gamma_k = 0, s_k) \) which combines packet loss and channel state, then the critical matrix is indeed the probability transition matrix for the joint state \( \tilde{s}_k \). \( \sigma_2 \rho(A)^2 \) can be viewed as a measure of how frequently the measurement packets get lost, while \( \rho(A) \) is a measure of how fast the system evolves [33].

For high dimensional systems, the matrix \( C \) of a single sensor may be a “wide” matrix and may not have full column rank. However, our results still can be valid in some practical cases. For example, if the sensor under concern is the head of a cluster of sensors, it can gather measurements from them and report the total information to the estimator. In this case, the whole observation matrix \( C \) will have enough rows to have full column rank. For general \( C \), it is mathematically difficult to derive necessary and sufficient conditions for the MSE stability. However, for a class of non-degenerate systems, one can follow the methods in [14], [34] to derive the stability conditions. The results are omitted due to space limitations.

In the rest of this section, we assume that the quality of \( CH_1 \) in its idle state is better than that of \( CH_0 \), i.e., \( \ell_1 < \ell_0 \). If otherwise, the sensor better stays on \( CH_0 \), which reduces the system to the conventional single channel case. Also assume that \( C \) has full column rank for simplicity. We use superscript \( \dagger \) on the variables to indicate the case without CHANCE.
B. Stability Gain Analysis

Define the stability region \( \Omega \) such that \( \forall A \in \Omega \), the estimation process is MSE stable. According to Theorem 2, \( \Omega = \{ A | A p (A)^2 < 1 \} \). Define the stability region radius as \( | \Omega | = \max_{A \in \Omega} \{ |p(A)| \} \) and the stability gain as \( \eta = \frac{1}{| \Omega |} \). It is easy to see that \( | \Omega | = \frac{1}{\sqrt{\rho}} \) and \(| \Omega | = \frac{1}{\sqrt{\beta}} \) according to Theorem 2. The following theorem characterizes the stability gain for which the proof is omitted due to space limitations.

**Theorem 3**: The stability gain \( \eta \) is monotonically increasing as the correct detection probability \( p_d \) increases or the false detection probability \( p_f \) decreases. Moreover,

\[
\eta = \sqrt{\frac{\ell_0}{\sigma_2}} \leq \min \left\{ \frac{1}{\sqrt{1 - \beta}}, \sqrt{\frac{\ell_0}{\ell_1}} \right\}.
\] (8)

The theorem shows that, in order to expand the stability region (i.e., \( \eta > 1 \)), we must have \( \sigma_2 < \ell_0 \). Since \( \ell_1 < \ell_0 \), by Lemma 1, we know that \( \sigma_2 \) is monotonically decreasing as either \( p_d \) increases or \( p_f \) decreases. Therefore, the condition \( \sigma_2 < \ell_0 \) yields a lower (or an upper) bound of \( p_d \) (or \( p_f \)). The upper bound in (8) is achievable if \( CH_1 \) is a constant channel (i.e., it is always busy or idle) and channel sensing is perfect (i.e., \( p_d = 1, p_f = 0 \)). If \( CH_1 \) is always busy, the sensing results will always suggest to transmit on \( CH_0 \) (with loss rate \( \ell_0 \)) and thus \( \eta = 1 \); similarly, if \( CH_1 \) is constantly idle, \( CH_1 \) will always be used and \( \eta = \sqrt{\frac{\ell_0}{\ell_1}} \).

C. Performance Analysis

In this part, we analyze the estimation performances in terms of the mean estimation error covariance \( \mathbb{E}[\ell_P[k]] \). We assume that both \( \ell_0 \) and \( \sigma_2 \) are less than \( \frac{1}{\rho(A)\beta} \) since otherwise either \( \mathbb{E}[\ell_P[k]] \) or \( \mathbb{E}[\ell_P[k]] \) will grow to infinity based on Theorem 2, making the performance comparison trivial.

**Theorem 4**: If \( \psi_2(1 - \beta) + \psi_1(1 - \beta) < \ell_0 \) and the initial conditions satisfy \( P_0 = 0 \), then \( \forall k \geq 0, \mathbb{E}[\ell_P[k]] \leq \mathbb{E}[\ell_P[k]] \). Moreover, if \( \psi_2(1 - \beta) + \psi_1(1 - \beta) < \ell_0 \), the inequality is strict.

By (6), the condition \( \psi_2(1 - \beta) + \psi_1(1 - \beta) < \ell_0 \) is equivalent to \( p_f(1 - \ell_0) < p_d(1 - \ell_0) \), which yields an upper (or lower) bound of \( p_f \) (or \( p_d \)). For energy detection mentioned above, this condition can be satisfied by appropriately tuning the channel sensing time and detection threshold \([3] \). Hence, Theorem 4 reveals that, under a mild condition, the estimation performance is definitely improved by CHANCE. Note that the improvement is guaranteed in very step.

In general, even though the mean error covariance \( \mathbb{E}[\ell_P[k]] \) is upper bounded, it does not necessarily converge to a unique steady state. Consider two sequences: \( \{ P[k] | P[k] = (1 - \gamma_k)A P[k-1] + W(1 - \gamma_k), k \geq 0, P_0 = 0 \} \) and \( \{ P[k] | P[k] = (1 - \gamma_k)A P[k-1] + W(1 - \gamma_k), k \geq 0, P_0 = 0 \} \), where \( W(\cdot) \) and \( \ell_P(\cdot) \) are defined in Definition 1. Below we show that these sequences are upper and lower bounds of \( \{ P[k] \} \), and both of them converge to their unique steady states in mean sense. Therefore, these two sequences facilitate us to study the behavior of \( \mathbb{E}[\ell_P[k]] \) over an infinite horizon.

**Definition 1**: \( \forall q \in [0, \frac{1}{\rho(A)\beta}] \), let us define the following functions: \( G_A(q) = \sum_{k=0}^\infty q^k A^k A^k \), \( W(q) = (1 - q)A(C - R - C)^{-1} A' + Q \), and \( \bar{W}(q) = (1 - q)A(C - R - C)^{-1} A' + Q \), where the first function presents the unique solution of Lyapunov equation \( \mathbb{E}[\ell_P[k]] \). The rest two functions are useful in determining upper and lower performance bounds.

**Theorem 5**: \( \forall k \geq 0, P_k \leq \bar{P}_k \) and \( \mathbb{E}[P_k] \leq \mathbb{E}[P_k] \). Moreover,

\[
\lim_{k \to \infty} \text{Tr}(\mathbb{E}[P_k]) = \text{Tr}(\mathbb{E}[P_\infty])
\]

\[
\text{Tr}(\mathbb{E}[\bar{P}_k]) = \text{Tr}(\mathbb{E}[\bar{P}_\infty])
\]

\[
\text{Tr}(\mathbb{E}[\bar{P}_k]) = \text{Tr}(\mathbb{E}[\bar{P}_\infty])
\]

\[
\text{Tr}(\mathbb{E}[\bar{P}_k]) = \text{Tr}(\mathbb{E}[\bar{P}_\infty])
\]

\[
\text{Tr}(\mathbb{E}[\bar{P}_k]) = \text{Tr}(\mathbb{E}[\bar{P}_\infty])
\]

Moreover, \( \varphi_\infty = \frac{\text{Tr}(\mathbb{E}[\bar{P}_\infty])}{\text{Tr}(\mathbb{E}[\bar{P}_\infty])} \).

**Theorem 6**: If \( \sigma_2 < \ell_0 \) and \( P_0 = 0 \), \( \mathbb{E}[\ell_P[k]] < \mathbb{E}[\ell_P[k]] \), \( \mathbb{E}[\ell_P[k]] < \mathbb{E}[\ell_P[k]] \), and the worst case performance ratio is

\[
\varphi_\infty = \frac{\text{Tr}(\mathbb{E}[\bar{P}_\infty])}{\text{Tr}(\mathbb{E}[\bar{P}_\infty])} + (1 - \delta) \frac{\text{Tr}(\mathbb{E}[\bar{P}_\infty])}{\text{Tr}(\mathbb{E}[\bar{P}_\infty])}.
\]

Moreover, \( \varphi_\infty \) is a monotonically decreasing function of \( p_d \).

The bounds in (9) and (10) are generally conservative mainly because of the gap between \( W(\cdot) \) and \( \ell_P(\cdot) \) due to the difference between \( A(C - R - C)^{-1} A' \) and \( A(C - R - C)^{-1} A' \). However, since \( A(C - R - C)^{-1} - (Q - C - R - C)^{-1} A' \), if either the measurement noise covariance \( R \) is sufficiently small or the system noise covariance \( Q \) is large, the above gap shrinks and thus the bounds become tight. In the extreme case where \( R = 0 \), \( W(\cdot) = \ell_P(\cdot) = Q \). Thus, the two bounds coincide and the performance ratio can be determined. To illustrate this, consider a class of special scalar systems with \( R = 0 \). Both \( P_k \) and \( P_k \) converge, and if \( P_0 = 0 \), the performance ratio is \( \varphi_\infty = P_0 \). This shows that if \( \sigma_2 < \ell_0 \) the bias is clearly less than 1 if \( \sigma_2 < \ell_0 \).

IV. EXTENSION TO MULTI-LICENSED-CHANNEL CASE

In this section, we consider that the sensor can opportunistically access multiple licensed channels. We first update the critical matrix, based on which the stability conditions and estimation performance analysis are then updated.
A. The Critical Matrix $\Phi\Psi$

Define mapping $B_m(\cdot)$ as follows. $\forall b \in \{1, \ldots, 2^m\}$, there exists a unique binary vector $B_m(b) = [b_1, \ldots, b_m]$ such that $b = \sum_{j=1}^{m} 2^{j-1} b_j + 1$. In other words, $B_m(b)$ is the binary representation of $b - 1$. Obviously, the inverse mapping $B_m^{-1}(\cdot)$ also exists and is unique. For the $m$ licensed channels, define the channel state vector $s_k = [s_{1,k}, \ldots, s_{m,k}]$. Since each channel has two states, $s_k$ has totally $2^m$ possibilities, each of which corresponds to a unique number in $\{1, \ldots, 2^m\}$ via the mapping $B_m^{-1}(s_k)$. Define the channel state probability vector $p_k = [P_{1,k}, \ldots, P_{2^m,k}]$, where $P_{b,k} = \mathbb{P}\{s_k = B_m(b)\} = \prod_{i=1}^{m} \mathbb{P}\{s_{i,k} = [B_m(b)]_i\}$, $\forall b \in \{1, \ldots, 2^m\}$. Obviously, $\sum_{b=1}^{2^m} p_{b,k} = 1$. Based on $p_k$, we can define the new transition probability matrix $\Phi$ whose dimension is $2^m \times 2^m$. Due to the hypothesis of inter-channel independency, each entry of $\Phi$ is

$$\begin{align*}
[\Phi]_{ij} &= \mathbb{P}\{s_{k+1} = B_m(j) | s_k = B_m(i)\} \\
&= \prod_{l=1}^{m} \mathbb{P}\{s_{l,k+1} = [B_m(j)]_l | s_{l,k} = [B_m(i)]_l\},
\end{align*}$$

where $i, j \in \{1, \ldots, 2^m\}$. Thus, one can easily prove that

$$\Phi = \Phi_m \otimes \Phi_{m-1} \otimes \cdots \otimes \Phi_1 \triangleq \bigotimes_{i=1}^{m} \Phi_i,$$

where “$\otimes$” represents the Kronecker product.

For the sensing matrix $\Psi$ in the form $\text{Diag}\{\psi_1, \ldots, \psi_{2^m}\}$, where $\forall b \in \{1, \ldots, 2^m\}$, $\psi_b = \mathbb{P}\{\gamma_k = 0 | s_k = B_m(b)\}$. Denote $\ell^2_0$ as the packet loss rate on $CH_1$ when it is in state $s$; $\ell^2_1 = 1$ if $s = 1$ and $\ell^2_1 = \ell^2_0$ otherwise. Thus,

$$\psi_b = \mathbb{P}\{\gamma_k = 0 | s_{1,k}, \ldots, s_{m,k}\} = \mathbb{P}\{o_{1,k} = 0 | s_{1,k} = b_1\} \ell^2_0 + \mathbb{P}\{o_{1,k} = 1 | s_{1,k} = b_1\},$$

$$\times \mathbb{P}\{o_{2,k} = 0 | s_{2,k} = b_2\} \ell^2_0 + \mathbb{P}\{o_{2,k} = 1 | s_{2,k} = b_2\},$$

$$\times \cdots \times \mathbb{P}\{o_{m-1,k} = 0 | s_{m-1,k} = b_{m-1}\} \ell^2_0 + \mathbb{P}\{o_{m-1,k} = 1 | s_{m-1,k} = b_{m-1}\},$$

$$\times \mathbb{P}\{o_{m,k} = 0 | s_{m,k} = b_m\} \ell^2_0 + \mathbb{P}\{o_{m,k} = 1 | s_{m,k} = b_m\} \ell^2_0,$$

$$= \sum_{i=1}^{m} \left( \prod_{j=1}^{i-1} \mathbb{P}\{o_{j,k} = 1 | s_{j,k} = b_j\} \right) \mathbb{P}\{o_{i,k} = 0 | s_{i,k} = b_i\} \ell^2_0 + \mathbb{P}\{o_{i,k} = 1 | s_{i,k} = b_i\} \ell^2_0.$$

where the second equality is based on the definition of sensing sequence $Q_m$, $\mathbb{P}\{o_{i,k} = 0 | s_{i,k} = b_i\} = \mathbb{P}\{p_{d,i} = b_i = 0\}$ if $p_{d,i} = 0$ and to $p_{f,i}$ if $b_i = 1$. Thus, $\Psi$ and the critical matrix are obtained.

B. Stability Analysis

Theorem 7: For system (1) with the above channel sensing schedule $Q_m$, a necessary condition for the MSE stability of the estimation process applied with CHANCE is

$$\rho(\Phi \Psi) \rho(A) < 1.$$  

(15)

Moreover, (15) is also sufficient if $C$ has full column rank.

In the following, we assume that $C$ has full column rank for simplicity. As mentioned before, extensions to a special class of non-degenerate systems are possible but algebraically complicated and thus are omitted due to space limitations.

C. Performance Analysis

Theorem 8: Let $\ell_{\min} = \min\{\ell_0, \ell_1, \ldots, \ell_m\}$. With the sensing schedule $Q_m$, the stability gain $\eta$ satisfies

$$\eta = \sqrt{\frac{\ell_0}{\rho(\Phi \Psi) \rho(A)}} \leq \min \left\{ \frac{1}{\sqrt{\prod_{i=1}^{m} (1 - \beta_i)}}, \sqrt{\ell_{\min}} \right\},$$

(16)

Let $\ell^* \triangleq \max\{\ell_i^\Phi \Psi u, \forall i \in \{1, \ldots, 2^m\}\}$, where $\ell_i^\Phi \Psi u$ is a $2^m \times 1$ vector with $[\ell_i^\Phi \Psi u]_j = 1$ and $[\ell_i^\Phi \Psi u]_i = 0$ for all $j \neq i$, $u = [1, \ldots, 1]_{1 \times 2^m}$. Similar to Theorem 4, the following theorem establishes the relationship between $\text{Tr}(\mathbb{E}[P_k])$ and $\text{Tr}(\mathbb{E}[E_k])$ for multi-licensed-channel cases. We focus on the nontrivial cases that $\ell_0 < \frac{1}{m \lambda^2}$ and (15) are satisfied.

Theorem 9: If $\ell^* \leq \ell_0$ and the initial conditions satisfy $P_0 = P_0$, then $\forall k \geq 0$, $\text{Tr}(\mathbb{E}[P_k]) \leq \text{Tr}(\mathbb{E}[P_0])$.

Theorem 10: $\forall k \geq 0$, $P_k \leq P_k$ and $\mathbb{E}[P_k] \leq \mathbb{E}[P_k]$ where $\{P_k\}$ and $\{P_k\}$ are defined above Theorem 5. Moreover,

$$\text{Tr}(\mathbb{E}[P_{\infty}]) = \sum_{i=0}^{\infty} [\nu' \Phi \Psi]^i \mathbb{E}[\rho(\Phi \Psi)^i],$$

where $\nu$ is the stationary state of the vector $p_k$, $\nu = 1 - \nu' (I - \Phi \Psi) u$. If $P_0 = P_0$,

$$\varphi \leq \frac{\text{Tr}(\mathbb{E}[G\rho(\Phi \Psi)])}{\text{Tr}(\mathbb{E}[G\rho(\Phi \Psi)])}.$$  

(19)

Remark 5: We focus on the scenario with one unlicensed channel and multiple licensed channels; however, our results are applicable to a wide range of scenarios. For example, (1) $CH_0$ is Markovian: consider another channel sensing order which differs from $Q_m$ in that the sensor will directly transmit packet through $CH_m$ if $CH_{m-1}$ is sensed busy (i.e., $p_{f,m} = p_{d,m} = 1$). If we deem $CH_m$ as the original channel, Theorem 7 is directly applicable to that the original channel has Markovian packet losses. Moreover, suppose $m = 1$ and deem $CH_m$ as the original channel; the scenario that the original channel is Markovian and CHANCE is not applied can be viewed as a special single-licensed-channel case. Then, the estimation performance can be analyzed in the way shown in Section III, and the results can be used to update Theorems 8-10. (2) Sensing multiple unlicensed channels: if the state dynamics of the unlicensed channels can be modeled by homogeneous Markov chains as shown in Proposition 1, one can see that our results are seamlessly applicable. In particular, an i.i.d. channel model is a special Markov model with $\alpha + \beta = 1$; in this case, it can be easily seen that the proofs of our theorems are still valid, and hence the theorems hold when multiple i.i.d. unlicensed channels are sensed by CHANCE.

V. Simulations

In this section, we evaluate the performances of the proposed mechanism based on a CPS with the physical process described by (1). We adopt the same parameters as in [13]:

$$A = \begin{bmatrix} 1.25 & 0 \\ 1 & 1.1 \end{bmatrix}$$

and $Q = 20I_{2 \times 2}$. The sensor measurements
are described by (2), where $C = I_{2 \times 2}$, $R = 2.5I_{2 \times 2}$, and the sampling period $T = 1$. The average loss rate on the original channel $CH_0$ is $\ell_0 = 0.5$. On licensed channel $CH_1$, the average busy and idle periods are $\frac{1}{\omega_{11}} = 0.35$ and $\frac{1}{\omega_{10}} = 0.3$, respectively. By Lemma 2, we have $\alpha = 0.537$ and $\beta = 0.461$. The measurement packets transmitted through $CH_1$ during its idle state will experience a loss rate $\ell_1 = 0.05$. The correct and false detection probabilities are $p_d = 0.8$ and $p_f = 0.3$, respectively. Thus, the largest eigenvalue of the critical matrix $\Phi \Psi$ is $\sigma_2 \approx 0.415 < \ell_0 < \frac{1}{\rho(A)^{\star}} = 0.64$. Hence, the Kalman filter based state estimation is stable in mean square sense either with or without CHANCE. However, the two cases achieve different performance in terms of $\text{Tr}(\mathbb{E}[P_k])$. As shown in Fig. 3(a), CHANCE achieves much better estimation performance.

Fig. 3 shows the performance bounds obtained in Theorem 5. First, we observe that the bounds are tight in our simulation case. Without CHANCE, it is well known that the estimation error covariance diverges if $\ell_0$ is larger than $\frac{1}{\rho(A)^{\star}} = 0.64$. In contrast, with $CH_1$ and CHANCE, the critical value increases significantly, i.e., the requirement for estimation stability is relaxed. From Fig. 3(b) we observe that CHANCE improves the estimation performance when $\ell_0 \geq 0.34$. In fact, when $\ell_0 \geq 0.34$, both $\sigma_2 < \ell_0$ and $\psi_2(1 - \beta) + \psi_1 \beta \leq \ell_0$ are true. Hence, by Theorem 4 and Theorem 6, the estimation performance is guaranteed to be improved. However, the figure also shows that there is a performance degradation when $\ell_0 < 0.34$ such that the quality of $CH_0$ is much better than that of $CH_1$ (the packet loss rate on $CH_0$ is lower than that of jointly using $CH_0$ and $CH_1$). This can be solved by improving the channel sensing accuracy as implied by Theorem 6.

More licensed channels will provide higher opportunities for the sensor to successfully transmit its measurement packets. To illustrate this, we introduce $m$ licensed channels to the system and each of them has the same property as $CH_1$ mentioned above. The correct and false detection probabilities over them are also set the same. The performance bounds and performance ratio obtained in Theorem 10 are shown in Table II, where $m = 0$ indicates the case without CHANCE. We can observe that: 1) with CHANCE, the upper bound and the relaxed upper bounds are quite close in all cases. 2) The worst case performance ratio is less than 36.68%; in other words, the mean square estimation error is reduced by at least 63.32%. 3) The performance is obviously improved by introducing to CHANCE more licensed channels. Whereas, such performance improvement becomes less significant as $m$ increases. Therefore, if the cost (e.g., energy expenditure) for channel sensing and switching is a concern, the sensor only needs to sense a few channels before each transmission.

We also conduct simulations to compare CHANCE and the mechanism (which we call as RANDOM) proposed in [22]. With RANDOM, in each step, the sensor chooses the same channel to sense if the channel is found idle in previous step; otherwise, it randomly chooses a different licensed channel to sense. To be comparable with CHANCE, RANDOM is enhanced by adding the original channel in the way that the sensor transmits data on the current channel if it is sensed idle and on the original channel otherwise. From Fig. 4 (where $m = 3$), we can see that CHANCE outperforms RANDOM. This is because, in each step, RANDOM senses only one channel while CHANCE may sense more; hence CHANCE gets more opportunity for transmitting during the channels’ idle periods. Moreover, as expected, the estimation error is reduced monotonically as the correct detection probability increases in both mechanisms, which is shown in Fig. 4(b).

VI. CONCLUSION

We have studied the state estimation problem in a class of CPS with linear process state dynamics. Based on the cognitive radio technology, we propose the CHANCE mechanism for the sensor to opportunistically access licensed spectrum for data transmissions. We develop new necessary and sufficient conditions for the estimation stability in mean square sense. For a special class of systems, we show that the estimation stability region is expanded by CHANCE. We also obtain a pair of upper and lower bounds for the estimation performance, and prove that they are monotonically decreasing as the correct detection probability increases. Explicit mathematical expression for the worst case performance ratio is derived. We also extend our results to accommodate any number of licensed channels. Simulation results show that the estimation performance is dramatically improved by CHANCE.

APPENDIX

A. Proof of Proposition 1

Due to the Markov property of the PU’s activities on $CH_i$, the sequence $\{s_{i,k}\}_{k \geq 0}$ is a homogeneous Markov chain, and the steady distributions are $\mathbb{P}\{s_{i,k} = 1\} = \frac{\omega_{i1}}{\omega_{i1} + \omega_{i0}}$, and $\mathbb{P}\{s_{i,k} = 0\} = \frac{\omega_{i0}}{\omega_{i1} + \omega_{i0}}$, respectively. Consider the continuous-time channel state $s_i(t)$. According to Kolmogorov’s Backward Equations, $\frac{\partial \Phi_i(t)}{\partial t} = G \Phi_i(t)$, where $\Phi_i(t) \triangleq \mathbb{P}\{s_i(t) = \ell|s_i(0) = \ell\}$, and the generator matrix is $G = \begin{bmatrix} -\omega_{i0} & \omega_{i0} \\ \omega_{i1} & -\omega_{i1} \end{bmatrix}$ according to [35]. Then, $\Phi_i(T) = e^{GT}$, which finally yields (5).
B. Proof of Theorem 1

We first introduce several useful lemmas.

**Lemma 1:** \( \sigma_2 = \rho(\Phi \Psi) \) and \( \sigma_{1,2} = \frac{1}{2} [\psi_1(1 - \alpha) + \psi_2(1 - \beta) + \sqrt{(\psi_1(1 - \alpha) - \psi_2(1 - \beta))^2 + 4\psi_1\psi_2\alpha\beta}] \). Moreover, both \( \sigma_1 \) and \( \sigma_2 \) are monotonically increasing as either \( \psi_1 \) or \( \psi_2 \) increases. If \( \ell_1 \leq \ell_0, \ell_1 \leq \ell_0 \leq \ell_2 \leq 1 \).

The following lemma models the packet loss process under CHANCE. \( \forall k \in 0 \land \forall k \geq 1 \), denote successive packet losses as \( \gamma(1, k) = 0 \) which means \( \{\gamma_k = 0, \gamma_{k+1} = 0, \ldots, \gamma_k = 0\} \). Define matrix \( M_k \) with each entry \( [M_k]_{ij} = \mathbb{P}\{s_k = j - 1, \gamma(k, k) = 0, s_0 = i\} \).

**Lemma 2:** \( M_k = (\Phi \Psi)^k, \forall k \geq 0 \). Moreover, \( \forall k \in 0 \) and \( \forall k \geq 1 \), \( \exists \varepsilon_1, \varepsilon_2 \subset (-\varepsilon_1, \varepsilon_1) \) and \( \varepsilon_2, \varepsilon_2 \subset \varepsilon_2, \varepsilon_2 \) such that the successive packet loss probability is \( \mathbb{P}\{\gamma_k = 0\} = \varepsilon_1, \varepsilon_2^{k+1} - 1 + \varepsilon_2, \varepsilon_2 \), where \( \varepsilon_1, \varepsilon_2 \) are all finite positive numbers.

**Proof:** \( \forall k > 0 \), from the definition of matrix \( M_k \),

\[
[M_k]_{11} = \mathbb{P}\{s_k = 0, s_{k-1} = 0, \gamma(k, k) = 0 | s_0 = 0\} + \mathbb{P}\{s_k = 0, s_{k-1} = 1, \gamma(k, k) = 0 | s_0 = 0\} = \mathbb{P}\{\gamma_k = 0, s_k = 0 | s_{k-1} = 0, \gamma(k, k) = 0, s_0 = 0\} \times [M_{k-1}]_{11} + \mathbb{P}\{\gamma_k = 0, s_k = 0 | s_{k-1} = 1, \gamma(k, k) = 0, s_0 = 0\} \times [M_{k-1}]_{12} = \psi_1 [M_{k-1}]_{11} + \psi_1 [M_{k-1}]_{12}.
\]

where the third equality is because of the Markov property of the channel state. The other three entries of \( M_k \) can be similarly obtained. Then, we have \( M_k = M_{k-1} \Phi \Psi \). By definition, \( M_0 = I \). Hence, \( M_k = (\Phi \Psi)^k \). Moreover, there exists an invertible matrix \( F_{\sigma} \) such that \( \Phi \Psi = F_{\sigma} \Lambda_\sigma F_{\sigma}^{-1} \), where \( \Lambda_\sigma = \text{Diag}(\sigma_1, \sigma_2) \). Therefore, \( M_k = F_{\sigma} (\Phi \Psi)^k F_{\sigma}^{-1} \).

Define the channel state probability vector \( p_k = [p_{0,k}, p_{1,k}] \), where \( p_{0,k} = \mathbb{P}\{s_k = b | b \in \{0, 1\}\} \). Then, the successive packet loss probability can be written as \( \mathbb{P}\{\gamma(1, k) = 0\} = \mathbb{P}\{\gamma(k, k) = 0 | s_{k-1} = 1\} p_{1,k-1} + \mathbb{P}\{\gamma(k, k) = 0 | s_{k-1} = 0\} p_{0,k-1} \).

Thus,

\[
\mathbb{P}\{\gamma(1, k) = 0\} = \mathbb{P}\{\gamma(1, k + 1 - k) = 0 | s_{0} = 0\} p_{1,k-1} + \mathbb{P}\{\gamma(1, k + 1 - k) = 0 | s_{0} = 0\} p_{0,k-1} = (M_{k+1-k+1} + [M_{k+1-k+1}][k+1-k]) p_{1,k-1} + \mathbb{P}\{\gamma(k, k) = 0 | s_{k-1} = 1\} p_{1,k-1} + \mathbb{P}\{\gamma(k, k) = 0 | s_{k-1} = 0\} p_{0,k-1}.
\]

**Lemma 3:** \( \forall A \in \mathbb{R}^{n \times n} \) and \( \forall k > 0 \), there exist \( \gamma_1 > 0 \) and \( \gamma_2 > 0 \) such that \( \gamma_1 \rho(A)^{2k} < \gamma_2 \rho(A)^{2k} \). \( \forall \sigma > 0 \) and \( \forall S \subset \mathcal{G} \) where \( \mathcal{G} \) is defined in Section II-A, \( \exists \gamma_3, \gamma_4 > 0 \) such that \( \gamma_3 \rho(A)^{2k} < \gamma_4 \rho(A)^{2k} \), where \( \gamma_4 > 0 \) and \( |\gamma| = \min |\gamma| > 0 \).

Let \( \gamma_1 = \lambda_{\min}(F_{\sigma} A F_{\sigma}) \lambda_{\min}(A F_{\sigma}^{-1} A F_{\sigma}^{-1}) = 0 \) and \( \gamma_2 = \lambda_{\max}(F_{\sigma} A F_{\sigma}) \lambda_{\max}(A F_{\sigma}^{-1} A F_{\sigma}^{-1}) = 0 \), then \( \text{Tr}(A^{2k} F_{\sigma} A F_{\sigma}) = \text{Tr}(F_{\sigma} A F_{\sigma}^{-1} A F_{\sigma}^{-1} F_{\sigma} A F_{\sigma}) = 0 \).

Then, \( \text{Tr}(A^{2k} F_{\sigma} A F_{\sigma}) = \text{Tr}(F_{\sigma} A F_{\sigma}^{-1} A F_{\sigma}^{-1} F_{\sigma} A F_{\sigma}) = 0 \).

Let \( \bar{s}_k = \lambda_{\min}(F_{\sigma} A F_{\sigma}) \lambda_{\min}(A F_{\sigma}^{-1} A F_{\sigma}^{-1}) = 0 \) and \( \bar{c}_k = \lambda_{\max}(F_{\sigma} A F_{\sigma}) \lambda_{\max}(A F_{\sigma}^{-1} A F_{\sigma}^{-1}) = 0 \), then \( \text{Tr}(A^{2k} F_{\sigma} A F_{\sigma}) = \text{Tr}(F_{\sigma} A F_{\sigma}^{-1} A F_{\sigma}^{-1} F_{\sigma} A F_{\sigma}) = 0 \).

Let \( \gamma_1 = \lambda_{\min}(F_{\sigma} A F_{\sigma}) \lambda_{\min}(A F_{\sigma}^{-1} A F_{\sigma}^{-1}) = 0 \) and \( \gamma_2 = \lambda_{\max}(F_{\sigma} A F_{\sigma}) \lambda_{\max}(A F_{\sigma}^{-1} A F_{\sigma}^{-1}) = 0 \), then \( \text{Tr}(A^{2k} F_{\sigma} A F_{\sigma}) = \text{Tr}(F_{\sigma} A F_{\sigma}^{-1} A F_{\sigma}^{-1} F_{\sigma} A F_{\sigma}) = 0 \).

**Lemma 4:** \( \forall A \in \mathbb{R}^{n \times n} \) and \( \forall S \subset \mathcal{G} \), under the packet loss process \( \gamma_k \geq 0 \) described in Section II-A, the sequence \( X_k | X_k = (1 - \gamma_k) A X_{k-1} + S, \forall k \geq 0, X \in \mathcal{G} \) is stable in mean sense iff. \( \gamma_2 \rho(A)^{2k} < 1 \).

**Proof:** \( \forall k > 0 \), we have \( X_k = (1 - \gamma_k) A X_{k-1} + S = \sum_{i=1}^{k} \sum_{i=1}^{k-1} (1 - \gamma_i) A X_i A \).

Taking expectation at both sides yields

\[
N_k = \frac{1}{\gamma_2} \frac{\gamma_1}{\gamma_2} \gamma_2 \rho(A)^{2k} \left| \gamma_3 \rho(A)^{2k} \right| = 0 \geq 0 \text{ is stable in mean sense iff. } \gamma_2 \rho(A)^{2k} < 1.
\]

Table II

<table>
<thead>
<tr>
<th>Number of licensed channels ( m )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower bound ( \text{Tr}(F_{\sigma} P_{\ell, k}) )</td>
<td>847.8</td>
<td>906.27</td>
<td>215.93</td>
<td>187.10</td>
<td>175.64</td>
<td>170.65</td>
<td>168.39</td>
<td>167.35</td>
<td>166.86</td>
</tr>
<tr>
<td>Upper bound ( \text{Tr}(F_{\sigma} P_{\ell, k}) )</td>
<td>857.89</td>
<td>910.78</td>
<td>219.37</td>
<td>190.18</td>
<td>178.57</td>
<td>173.52</td>
<td>171.23</td>
<td>170.17</td>
<td>169.68</td>
</tr>
<tr>
<td>Worst case performance ratio by (19)</td>
<td>100%</td>
<td>36.68%</td>
<td>25.90%</td>
<td>22.45%</td>
<td>20.68%</td>
<td>20.24%</td>
<td>20.21%</td>
<td>20.09%</td>
<td>20.03%</td>
</tr>
</tbody>
</table>
\[ \mathbb{E}[X_k] = \sum_{i=1}^{k-1} \mathbb{P}\{\gamma(k+1-i, k) = 0\} A^k \sigma_1^i S A^i \]
\[ + \mathbb{P}\{\gamma(1, k) = 0\} A^k X_0 A^k + S \]
\[ = \sum_{i=1}^{k-1} (\xi_1, k+1-i, \sigma_1^i + \varepsilon_2, k+1-i, \frac{\sigma_1^j}{2}) A^k \sigma_1^i S A^i \]
\[ + (\xi_1, \sigma_1^0 + \varepsilon_2, \sigma_1^0) A^k X_0 A^k + S \]  
(24)

1) Sufficiency: By Lemma 2, we have \( \text{Tr}(\mathbb{E}[X_k]) \leq \text{Tr}(S) + \sum_{i=1}^{k-1} \mathbb{E}[\gamma_2^2, \sigma_2^2] \mathbb{E}[A^i \sigma_2^0 S A^i] \) \( \leq \text{Tr}(S) + \sum_{i=1}^{k-1} \mathbb{E}[\sigma_2^2 A^i S A^i] \)
\( \leq \text{Tr}(S) + \sum_{i=1}^{k-1} \mathbb{E}[\sigma_2^2 A^i S A^i] + \text{Tr}(X_0) k^2 \sigma_2^2 (\sigma_2(\rho(A)^2)^2) \).
If \( \sigma_2(\rho(A)^2)^2 < 1 \), \( \exists \varepsilon \in (\sigma_2(\rho(A)^2), 1) \) such that \( k^{2n} < \frac{\varepsilon^2}{\varepsilon^2 + \alpha^2} (k - 1)^{2n} \) holds for all \( k > k_0 \).

2) Necessity: Above we know that \( \varepsilon^2, k \geq \varepsilon_2 > 0 \) and also \( \sigma_2 \geq \sigma_1 \geq 0 \). Hence, \( \exists \varepsilon > 0 \) such that \( \forall k > k_0 \), \( -\xi_1 \frac{\sigma_1}{\sigma_2} k + \xi_2 > \varepsilon ' \). Hence, \( \text{Tr}(\mathbb{E}[X_k]) \geq \sum_{i=1}^{k-1} \mathbb{E}[\gamma_2^2, \sigma_2^0] (\xi_1, k+1-i, \xi_2, k+1-i, \sigma_2^0) \mathbb{E}[A^i \sigma_2^0 S A^i] \geq \sum_{i=1}^{k-1} (-\xi_1 \sigma_1 + \xi_2 \sigma_2) \mathbb{E}[A^i \sigma_2^0 S A^i], \)
\( \sum = \sum_{i=1}^{k-1} (-\xi_1 \sigma_1 + \xi_2 \sigma_2 - \varepsilon ' \sigma_2) \mathbb{E}[A^i \sigma_2^0 S A^i] - \mathbb{E}[\gamma_2^2, \sigma_2^0] \mathbb{E}[A^i \sigma_2^0 S A^i] \).
\( \sum > \mathbb{E}[\gamma_2^2, \sigma_2^0] \mathbb{E}[A^0 \sigma_2^0 S A^0] \).
\( \sum > 0 \).
\( \mathbb{E}[X_k] > 0 \).
\( \alpha \geq 0 \).

The predicted error covariance \( P_k \) is lower bounded by [13]
\[ P_k \geq (1 - \gamma_k) A P_{k-1} A' + Q. \]  
(25)

Then, based on Lemma 4, Theorem 1 can be proved.

C. Proof of Theorem 2
First, it is easy to verify that \( (P_{k-1} + C' R^{-1} C)^{-1} C' R^{-1} = P_{k-1} C' (P_{k-1} C + R)^{-1} \).
Then, after simple matrix manipulations, \( P_k \) can be written as \( (1 - \gamma_k) P_{k-1} A' + \gamma_k A (P_{k-1} + C' R^{-1} C)^{-1} A' + Q. \)
\( \gamma_k \geq 0 \). If \( C \) has full column rank, \( (P_{k-1} + C' R^{-1} C)^{-1} \leq (C' R^{-1} C)^{-1} \leq (C' R^{-1} C)^{-1} < \infty \) since \( P_k > 0 \).
Therefore,
\[ P_k \leq (1 - \gamma_k) A P_{k-1} A' + W \]  
(26)
with \( W = A (C' R^{-1} C)^{-1} A' + Q. \) By Lemma 4, \( \sigma_2(\rho(A)^2)^2 < 1 \) is a sufficient condition for MSE stability of the Kalman filter. Combined with Theorem 1, the proof is thus completed.

D. Proof of Theorem 4
Define following two maps \( L, R : S_+^n \rightarrow S_+^n \).
\( L(X) = A X A' + Q \) and \( R(X) = A X A' + Q - A X C (C X A')^{-1} A' \).
\( \forall \phi \geq 0 \), by definitions of \( \psi_{1, 2}, \alpha \) and \( \beta \),
\[ \mathbb{E}[X_k] \leq \sum_{i=1}^{k-1} \mathbb{P}\{\gamma(k+1-i, k) = 0\} A^k \sigma_1^i S A^i \]
\[ + \mathbb{P}\{\gamma(1, k) = 0\} A^k X_0 A^k + S \]
\[ = \sum_{i=1}^{k-1} (\xi_1, k+1-i, \sigma_1^i + \varepsilon_2, k+1-i, \frac{\sigma_1^j}{2}) A^k \sigma_1^i S A^i \]
\[ + (\xi_1, \sigma_1^0 + \varepsilon_2, \sigma_1^0) A^k X_0 A^k + S \]  
(24)

Since \( \alpha + \beta < 1 \) by Remark 1 and \( \psi_1 \leq \psi_2 \) by Lemma 1,
\( \psi_1 (1 - \alpha) + \psi_2 \alpha \geq \psi_2 \).
\( \psi_1 \leq \psi_2 \).
\( \psi_2 (1 - \beta) + \psi_1 \beta \geq \psi_0 \).

We now apply an induction argument to show that \( \mathbb{P}\{P_k > \phi I\} \leq \# \mathbb{P}\{P_k \leq \phi I\} \).
Clearly, it holds for \( k = 0 \). Suppose it holds for \( k > 0 \), then, \( \mathbb{P}\{P_{k+1} > \phi I\} \leq \# \mathbb{P}\{P_k < \phi I\} \).
We prove that \( \mathbb{P}\{P_{k+1} > \phi I\} \leq \# \mathbb{P}\{P_k < \phi I\} \).
\( \mathbb{P}\{P_k > \phi I\} \geq \# \mathbb{P}\{P_k < \phi I\} \).
\( \mathbb{P}\{P_{k+1} > \phi I\} \leq \# \mathbb{P}\{P_k < \phi I\} \).

E. Proof of Theorem 5
We first introduce two lemmas, following which the proof of Theorem 5 is straightforward. \( \forall W \in S_+^n \) and \( \forall q \in [0, \frac{1}{\sqrt{\pi r^2}}] \), define \( \mathcal{H}_A(q, W) = \sum_{i=0}^{r-1} q_i A^i W(A^i)' \).
Obviously, \( \mathcal{H}_A(q, W) = \mathcal{H}_A(q, I) \), where \( \mathcal{H}_A(q) \) is defined in
\[ \mathcal{H}_A(q, W) = \sum_{i=0}^{k-1} q_i A^i W(A^i)' \]  
(25)

\begin{align*}
\mathcal{H}_A(q, W) & \leq \mathcal{H}_A(q, I) \leq \mathcal{H}_A(q, W) \\
\mathcal{H}_A(q, W) & \leq \mathcal{H}_A(q, I) \leq \mathcal{H}_A(q, W)
\end{align*}
Lemma 2. By induction, \( \frac{d}{dq} X \) is finite. It can be seen that \( X = q(A \bar{X} A' + (1-q)W) \) gives \( \bar{X} = W = q(A \bar{X} W + q(A^2W - W)) \). Since \( A^2W - W > 0, \bar{X} - W > 0 \) and hence \( A \bar{X} - A' > 0 \). Taking derivatives at both sides of the above Lyapunov equation for \( \bar{X} \), we have \( \frac{d}{dq} X = q(A \bar{X} A' + A^2W - W) \). Therefore, \( \frac{d}{dq} X_k = \sum_{k=0}^{\infty} \delta_k A_k (A \bar{X} A' - A^2W) A_k > 0 \), which proves the monotonicity of \( X_k \).

Lemma 6: For the above sequence \( \{X_k\} \), we have: (1) \( \{X_k\} \) is stable in mean sense iff. \( \sigma_2 \rho(A)^2 < 1 \). (2) If \( \sigma_2 \rho(A)^2 < 1 \), \( E[X_k] \) converges to a unique value \( E[X_{\infty}] \), with \( \lim \sup \{E[X_k]\} \leq (1-\delta) \lim \sup \{E[X_{k-1}]\} A^4S A^4 + \delta \sum_{i=0}^{k-1} E[X_{k-i+1}]A^4S A^4 \). (3) If \( \sigma_2 \rho(A)^2 > 1 \), \( p_k > 0 \) increases, \( \lim \sup \{E[X_k]\} < (1-\delta) \lim \sup \{E[X_{k-1}]\} A^4S A^4 + \delta \sum_{i=0}^{k-1} E[X_{k-i+1}]A^4S A^4 \). (4) If \( \sigma_2 \rho(A)^2 = 1 \), \( p_k \) converges to 0.

Proof: (4) Since \( q \in [0, \frac{1}{\rho(A^2)}] \), the first property is obvious. (2) Let \( \bar{X} = (1 - q)H_A(q, W) \). First, by a similar technique as in the proof of Lemma 4, we can show that
\[
E[X_k] = E[(1 - q)AX_{k-1} + q W] + q W > 0, X_0 \in S_Q.
\]
(3) If \( \sigma_2 \rho(A)^2 < 1 \), \( E[X_k] \) converges to a unique value \( E[X_{\infty}] \), with
\[
\lim \sup \{E[X_k]\} \leq (1-\delta) \lim \sup \{E[X_{k-1}]\} A^4S A^4 + \delta \sum_{i=0}^{k-1} E[X_{k-i+1}]A^4S A^4 \].

Proof: Define matrix \( M_k \in \mathbb{R}^{2m \times 2m} \), where \( M_k = B_m(i) \). Following the very similar way as in (20), we can prove that
\[
\{E[X_k]\} \leq (1-\delta) \lim \sup \{E[X_{k-1}]\} A^4S A^4 + \delta \sum_{i=0}^{k-1} E[X_{k-i+1}]A^4S A^4.
\]
(2) If \( \sigma_2 \rho(A)^2 > 1 \), \( p_k \) increases,
\[
\lim \sup \{E[X_k]\} < (1-\delta) \lim \sup \{E[X_{k-1}]\} A^4S A^4 + \delta \sum_{i=0}^{k-1} E[X_{k-i+1}]A^4S A^4.
\]
(4) If \( \sigma_2 \rho(A)^2 = 1 \), \( p_k \) converges to 0.
where the first inequality is based on the Cauchy-Schwarz inequality, while the last one is based on Lemma 3. Therefore,

\[
\begin{align*}
\text{Tr}(E[X_k]) &= \mathbb{P}\{\gamma(1, k) = 0\} \text{Tr}(A^k X_0 A^k) \\
&\quad + \sum_{k=1}^{m} \mathbb{P}\{\gamma(k, k + 1 - i) = 0\} \text{Tr}(A^i X_0 A^k) + \text{Tr}(S) \\
&< 2\bar{\zeta} c_2^m k 2^n \rho (\Phi \Psi)^k \text{Tr}(A^k X_0 A^k) \\
&\quad + 2\bar{\zeta} c_2^m \sum_{i=1}^{k-1} \rho (\Phi \Psi)^{i} \text{Tr}(A^i X_0 A^k) + \text{Tr}(S) \\
&< 2\bar{\zeta} c_2^m \text{Tr}(X_0) k 2^{m+2n} \rho (\Phi \Psi)^k \rho (A)^{2k} \\
&\quad + 2\bar{\zeta} c_2^m \text{Tr}(S) \sum_{i=1}^{k-1} \rho (\Phi \Psi)^{i} \rho (A)^{2i} + \text{Tr}(S).
\end{align*}
\]

Similar to the proof of Lemma 4, a sufficient condition for \(\sup_k \text{Tr}(E[X_k]) < \infty\) is \(\rho (\Phi \Psi) \rho (A) < 1\).

2) Necessity: Because \(\alpha_i + \beta_i < 1\), \(\mathbb{P}\{s_i, k+1 = 1\} = 1 - \beta_i \mathbb{P}\{s_i, k = 1\} + \alpha_i \mathbb{P}\{s_i, k = 0\} = (1 - \alpha_i - \beta_i) \mathbb{P}\{s_i, k = 1\} + \alpha_i \mathbb{P}\{s_i, k+1 = 1\} \geq \beta_i\), and also \(\mathbb{P}\{s_i, k+1 = 0\} \geq \beta_i\). Therefore, \(\exists \beta_{\min} > 0\) such that \(\forall k \geq 1\), \(p_{b,k} \geq \prod_{i=1}^{m} \min \{\alpha_i, \beta_i\} = p_{\min}\). Then,

\[
\begin{align*}
\mathbb{P}\{\gamma(k_1, k_2) = 0\} &\geq p_{\min} \sum_{b=1}^{B} \sum_{l=1}^{B} |M_{k_2+1-k_1}|_{lb} \\
&\geq \sqrt{p_{\min} \sum_{b=1}^{B} \sum_{l=1}^{B} |M_{k_2+1-k_1}|_{lb}^2} \\
&= \sqrt{p_{\min} \text{Tr}(M_{k_2+1-k_1} M_{k_2+1-k_1}^\dagger)} \geq p_{\min} \zeta_1 (\Phi \Psi)^{k_2+1-k_1},
\end{align*}
\]

where \(\zeta_1 > 0\) and the last inequality is from Lemma 3. Then, the necessity is obvious.

In view of the above lemma, Theorem 7 can be similarly proved based on (25) and (26) which have been used to prove Theorems 1 and 2.

H. Proof of Theorem 8

Lemma 8 ([36]): For any two square matrices \(X, Y \in \mathbb{R}^{n \times n}\), \(\rho(X) \geq \rho(Y)\) if for all possible \(i, j\), \([X]_{ij} \geq [Y]_{ij} \geq 0; \rho(X \otimes Y) = \rho(X) \rho(Y)\).

\(\forall b \in \{1, \ldots, 2^n\}\), if \(b_m = 1\), from (14) we have

\[
\begin{align*}
\psi_b &= [p_{f,m} + \ell_0 (1 - p_{f,m})] \prod_{j=1}^{m-1} \mathbb{P}\{o_{j,k} = 1 | s_{j,k} = b_j\} \\
&\quad + \sum_{i=1}^{m-1} \prod_{j=1}^{i-1} \mathbb{P}\{o_{j,k} = 1 | s_{j,k} = b_j\} \mathbb{P}\{o_{i,k} = 0 | s_{i,k} = b_i\} \epsilon_i^b \\
&\quad + \ell_0 \prod_{j=1}^{m-1} \mathbb{P}\{o_{j,k} = 1 | s_{j,k} = b_j\},
\end{align*}
\]

where the equality holds only when \(p_{f,m} = 0\). If \(b_m = 0\), in a similar way, we have

\[
\begin{align*}
\psi_b &\geq m^{-1} \sum_{i=1}^{m-1} \left( \prod_{j=1}^{i-1} \mathbb{P}\{o_{j,k} = 1 | s_{j,k} = b_j\} \right) \mathbb{P}\{o_{i,k} = 0 | s_{i,k} = b_i\} \epsilon_i^b \\
&\quad + \min(\ell_0, \ell_m) \prod_{j=1}^{m-1} \mathbb{P}\{o_{j,k} = 1 | s_{j,k} = b_j\},
\end{align*}
\]

and the equality holds only when \(p_{d,m} = 0\) if \(\ell_0 < \ell_m\) or \(p_{d,m} = 1\) if \(\ell_0 \geq \ell_m\). Continuing applying this method to \(\{b_{m-1}, \ldots, b_1\}\), we finally get that \(\psi_b \geq \ell_0\) if \(b = 2^m\) and \(\psi_b \geq \ell_{\min}\) otherwise. Thus, \(\rho (\Phi \Psi) \geq \rho (\Phi \text{Diag}\{0, \ldots, 0, \ell_0\}) = \ell_0 \prod_{i=1}^{m} (1 - \beta_i), \) if \(\ell_{\min} = 0\). In this case, the stability gain \(\eta \leq \frac{1}{\sqrt{\ell_0 (1 - \beta_i)}}\). Otherwise, if \(\ell_{\min} > 0\), by Lemma 8, we have \(\rho (\Phi \Psi) \geq \rho (\rho (\Phi) \Phi) = \rho (\Phi) \rho (\Phi) = \rho (\Phi) = \rho (\Phi) \rho (\Phi) = \rho (\Phi)\). Thus, \(\eta \leq \frac{1}{\sqrt{\ell_{\min}}}\). In sum, (16) holds and thus the theorem is proved.

I. Proof of Theorem 9

Similar to the proof of Theorem 4, we have

\[
\begin{align*}
P\{\text{P}_{k+1} > \phi I\} &= P\{\mathcal{L}(P_{k}) > \phi I, \gamma_{k} = 0\} \\
&\quad + P\{\mathcal{R}(P_{k}) > \phi I, \gamma_{k} = 1\} \\
&= \sum_{b=1}^{2^m} \left[ P\{\mathcal{L}(P_{k}) > \phi I, s_{k-1} = B_{m}(b)\} \sum_{i=1}^{2^m} [M_{i}]_{bi} \\
&\quad + P\{\mathcal{R}(P_{k}) > \phi I, s_{k-1} = B_{m}(b)\} \sum_{i=1}^{2^m} [\Phi]_{bi} - [M_{i}]_{bi} \right] \\
&= \sum_{b=1}^{2^m} \left[ P\{\mathcal{L}(P_{k}) > \phi I, s_{k-1} = B_{m}(b)\} \sum_{i=1}^{2^m} [\Phi]_{bi} - [M_{i}]_{bi} \right] \\
&\quad + \epsilon_\Phi \Phi \psi_u \left[ P\{\mathcal{L}(P_{k}) > \phi I, s_{k-1} = B_{m}(b)\} \\
&\quad - P\{\mathcal{R}(P_{k}) > \phi I, s_{k-1} = B_{m}(b)\} \right] \\
&\leq P\{\mathcal{R}(P_{k}) > \phi I\} + \tilde{\ell} \sum_{b=1}^{2^m} P\{\mathcal{L}(P_{k}) > \phi I, s_{k-1} = B_{m}(b)\} \\
&\quad - P\{\mathcal{R}(P_{k}) > \phi I\} + (1 - \tilde{\ell}) \sum_{b=1}^{2^m} P\{\mathcal{L}(P_{k}) > \phi I\},
\end{align*}
\]

where we have used the fact that \(\sum_{i=1}^{2^m} [\Phi]_{bi} = 1, \forall b \in \{1, \ldots, 2^m\}\). In the same way as in the proof of Theorem 4, the proof of the current theorem can be achieved.

J. Proof of Theorem 10

In the proof of Lemma 6, we show that \(\forall k \geq 0, p_{0,k} = (p_{0,0} - \frac{\beta}{\alpha + \gamma}) z^{k} + \frac{\beta}{\alpha + \gamma} \), which can be generalized to the multi-licensed-channel case as \(P\{s_{i,k} = s\} = \xi_{i,s} + \xi_{i,s} z_{i}^{k}\), where \(s \in \{0, 1\}, \xi_{i,s}\) and \(\xi_{i,s}\) are constants. Thus, \(\forall b \in \{1, \ldots, 2^m\}\), \(p_{b,k} = P\{s_{k} = B_{m}(b)\} = \prod_{i=1}^{m} \xi_{i,[s_{i},B_{m}(b)]} + \xi_{i,[s_{i},B_{m}(b)]} z_{i}^{k}\), where \(w_{b} = \prod_{i=1}^{m} \xi_{i,[B_{m}(b)]}, \)
\[ z \triangleq \max\{z_1, \ldots, z_m\} \]. The expression of \( \tilde{w}_{b,k} \) is complicated but it is easy to see that, \( \tilde{w}_{b,k} \) is above bounded by some \( \tilde{w}_b \). Since \( z \in (0,1) \), \( \tilde{w}_b \) is \( \lim_{z \to \infty} \tilde{w}_{b,k} \) Thus, the channel state vector can be rewritten as \( \tilde{p}_k = \tilde{v} + \tilde{v}_k z^k \), where \( \tilde{v}_k = [\tilde{v}_k, \ldots, \tilde{v}_{2m-k}]^T \). Let \( \tilde{v} = [\tilde{v}_1, \ldots, \tilde{v}_{2m}]^T \). As shown in the proof of Lemma 7, the consecutive packet loss probability is \( P(\gamma_k, \tilde{v}_2) = 0 \). \( \tilde{p}_{k+1}^T (\Phi \Psi)^{k+1} u = (\tilde{v} + \tilde{v}_{k+1} z^{k+1} - 1)^T (\Phi \Psi)^{k+1} u \).

Then, according to (28), the mean of the sequence \( \{X_k\} \) becomes

\[
E[X_k] = (\tilde{v} + \tilde{v}_{k+1} z^{k+1} - 1)^T (\Phi \Psi)^{k+1} u A^i Q A^i
\]

If \( \rho (\Phi \Psi) \rho (A) < 1 \), \( (\Phi \Psi)^{k+1} u \rightarrow 0 \). Since \( \Phi \Psi \leq \rho (\Phi \Psi) I \), \( (\Phi \Psi)^{k+1} u \leq \tilde{v}_{k+1} z^{k+1} - 1 \rho (\Phi \Psi) \leq \tilde{v} u z^{k+1} - 1 \rho (\Phi \Psi) \).

With \( \rho (\Phi \Psi) < \frac{1}{\rho (A)} \), by Lemma 5,

\[
\lim_{k \to \infty} \sum_{i=1}^{k} (\tilde{v}_{k+1} - 1 \rho (\Phi \Psi) A^i S A^i) = 0.
\]

Then, the lower and upper bounds as in (17) and (18) of the current theorem are evident.

Though (18) can be used to compute the worst case performance ratio in the way similar to Theorem 6, the computation may be very complicated as the critical matrix can be nearly in any form (may not be diagonalizable). Instead, we derive a relaxed but much simpler bound for \( \varphi_\infty \). Since \( \nu' (\Phi \Psi)^i u \leq \nu' \rho (\Phi \Psi)^i u \leq \nu' \rho (\Phi \Psi)^i u \leq \nu' (I - \Phi \Psi)^i u A^i Q (1 - 0) L W A^i \).

Then, the lower and upper bounds as in (17) and (18) of the current theorem are evident.

References


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