Capacity Gain through Power Enhancement in Multi-Radio Multi-Channel Wireless Networks

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***Abstract*—The main focus of this paper is to show theoretically that power is a crucial factor in multi-radio multi-channel (MR-MC) wireless networks and hence by judiciously leveraging the power, one can realize a considerable gain on the capacity for MR-MC wireless networks. Such a capacity gain through power enhancement is revealed by our new insights of a *co-channel enlarging effect*. In particular, when the number of available channels (c) in a network is larger than that necessary for enabling the maximum set of simultaneous transmissions ($\tilde{c}$), allocating transmissions to those additional $c - \tilde{c}$ channels could enlarge the distance between the co-channel transmissions; the larger co-channel distance then allows a higher transmission power for higher link capacity. The finding of this paper specifically indicate that by exploiting the co-channel enlarging effect with power, one can realize the following gain on the capacity for MR-MC wireless networks: (i) In the channel-constraint region ($\tilde{c} < c < \frac{n\phi}{2}$), if each node augments its power from the minimum P_{min} to $P_{min}\frac{c}{\tilde{c}}^{\frac{\alpha}{2}}$, then a gain of $\Theta(\log(\frac{c}{\tilde{c}})^{\frac{\alpha}{2}})$ is achieved; (ii) In the power-constraint region ($c \geq \frac{n\phi}{2}$), if each node sends at the maximum power level, $P_{max} = P_{min}.n^K$ or $P_{min}.2^{\frac{n\phi}{2}}$, depending on the power availability at a node, then a gain of $\Theta(\log n)$ or $\Theta(n)$ is achieved, respectively.**

I. Introduction

The existence of interference among wireless communications is one of the crucial challenges that affects the capacity of wireless networks, particularly in multi-hop settings. In their pioneering work [2], Gupta and Kumar have proved that when n nodes are randomly or arbitrarily deployed in a planar disk of unit area, the amount of information that can be exchanged by each source-destination pair becomes vanishingly small, as n grows to a large level and consequently, the scalability of such multi-hop wireless networks is undeniably affected. This dampening result in fact comes about as a result of the exclusion zones required for those interference based communication paradigms. As the area of exclusion zones directly depends on the transmit power, their work essentially propose to reduce the transmit power of nodes to as small a value as possible, without sacrificing the connectivity, to improve the capacity. Nevertheless, it turns out from the Shannon-Hartley theorem [1] that when nodes rely on lower transmit power the SNR decreases with the degraded quality of received signal and correspondingly, the channel capacity. A little amount of reflection shows that there are trade-offs between the two quantities, transmit power and wireless interference, in that one can only be improved at the expense of the other. In the last few years research efforts, [1], [6] and the references therein, indicate that equipping nodes with multiple radios and operating these radios on multiple frequency channels can greatly mitigate the negative effects of wireless interference and thus enhance the capacity of these networks. Since single channel wireless networks have no other alternatives but rely on minimum power due to the spatial concurrency constraints, it is interesting to see whether power enhancement can render further gain in multi-radio multi-channel (MR-MC) wireless networks. In [6], Kysanaur and Vaidya first studied the asymptotic capacity of MR-MC wireless networks, along the theme of *minimum transmit power*, with c orthogonal channels and ϕ radio interfaces per node. Their results have essentially proved that when the ratio $\frac{c}{\phi} = O(\log n)$, albeit the bandwidth is split into c channels, there is no degradation in the capacity when compared to the single-channel network and thus a per-node capacity of $\Theta(\sqrt{\frac{1}{n \log n}})$ bits/sec can be realized. However, as the ratio $\frac{c}{\phi}$ increases, of channels to radios becomes larger, the asymptotic capacity of each node decreases and ultimately, approaches zero when $c/\phi = \Omega(n)$. In this paper, by extending the analysis in [6] we show that one can realize extra capacity gain in MR-MC wireless networks by enhancing the power; the capacity can even be improved to $\Theta(1)$ in the region $c/\phi = \Omega(n)$ when exponential power enhancement is allowed.

The motivation behind this work comes from the fact that when the number of available channels (c) in a network is larger than that necessary for enabling the maximum set of simultaneous transmissions ($\tilde{c}$), allocating transmissions to those residual $c - \tilde{c}$ channels could enlarge the distance between co-channel transmissions. We term this novel insight as *co-channel enlarging effect*. The larger co-channel distance then allows a higher transmission power for higher link capacity. For instance, Fig. 1(a) shows the placement of interference (exclusion) zones around each sender-receiver pair communicating at the minimum power P_{min}. Assume $\tilde{c} = 3$, Fig 1(a) shows the scenario where each overlapping zone is allocated to a distinct channel so that all communications can happen concurrently. If there are $c = 5$ channels available, in Fig 1(b) we exploit the co-channel enlarging effect by relocating some transmissions on channel 1 to channels 4 and 5 such that the distance between the transmissions on channel 1 can be enlarged. The larger co-channel distance in turn enables each node to augment its power without disturbing the existing co-channel transmissions. Fig 1(c) shows the exploitation of co-channel enlarging effect, where each sender-receiver pair transmit at a power level $P > P_{min}$ for higher capacity, thus

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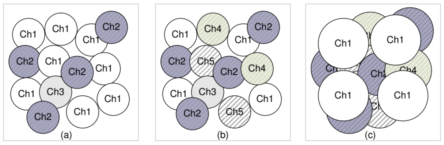

Fig. 1. (a) Original Network at P_{min}; (b) Co-Channel Enlarging effect; (c) Exploiting the Effect with Power $P > P_{min}$. We assume $\tilde{c} = 3$ and $c = 5$.

occupying a larger interference range but not impacting other simultaneous transmissions. Moreover, if there are enough number of channels so that every possible link in the network can be assigned a distinct channel, each sender node can then use its full power to achieve the maximum link capacity.

Motivated by such an idea, we particularly address the following two questions: (i) How much gain can one realize by exploiting the co-channel enlarging effect with power in MR-MC networks? and (ii) What is the optimal power assignment for nodes in a MR-MC network? Our paper answers these two questions by analyzing the asymptotic capacity bounds of MR-MC networks utilizing power in both arbitrary and random networks. We consider that each node is equipped with ϕ radio interfaces. The results derived in this paper stipulate that whether arbitrary or random network setting, by choosing appropriate node power levels in the following two regimes, $\tilde{c} < c < \frac{n\phi}{2}$ (where co-channel interference exists) and $c \geq \frac{n\phi}{2}$ (where each link has one distinct channel), one can realize a significant gain on the capacity for MR-MC wireless networks. The findings of this work particularly stipulate that: (a) In the *channel-constraint* region, $\tilde{c} < c < \frac{n\phi}{2}$, if each node augments its power from the minimum P_{min} to $P_{min}\frac{c}{\tilde{c}}^{\frac{\alpha}{2}}$, then a gain of $\Theta(\log(\frac{c}{\tilde{c}})^{\frac{\alpha}{2}})$ is achieved; (b) In the *power-constraint* region, $c \geq \frac{n\phi}{2}$, if each node exploits the maximum power level, $P_{max} = P_{min}.n^K$ or $P_{min}.2^{\frac{n\phi}{2}}$, depending on the power availability at a node, then a gain of $\Theta(\log n)$ or $\Theta(n)$ is achieved, respectively. It is noteworthy, the findings of this paper are of particular importance to the scenarios that have no energy and power constraints such as Wireless Mesh Networks (WMNs) consisting of infrastructure mesh routers. To summarize, the main contributions of our paper are as follows: (i) This work produces the first effort to propose the concept of *co-channel enlarging effect* to manifest analytically the benefits of enhancing power in MR-MC wireless networks; (ii) By extending the analysis in [6], we derive the asymptotic capacity bounds of the proposed model for both random and arbitrary network setting. (iii) Our paper mainly show that power is a critical factor and by optimally choosing the power assignments, one can realize a considerable gain on the capacity for MR-MC networks. **Note**: We particularly claim that given the maximum power available to nodes in a network, one could use the results of this work to determine the optimal number of nodes needed for constructing a capacity efficient network.

The rest of the paper is organized as follows. Section II presents some of the related works in the area of capacity analysis and power-control. In section III, we discuss the background pertaining to network model to facilitate the derivation of asymptotic bounds. Section IV discusses the main results of this paper. In section V we present some useful lemmas that is used to obtain the capacity bounds of the proposed solution under random and arbitrary settings in section VI. Finally, we summarize our work in section VII.

II. Related Work

We briefly discuss some of the related efforts in the area of capacity analysis and power-control. The dampening result of [2] motivated many researchers to consider several ways including mobility, hybrid architecture, unlimited bandwidth resources to increase the capacity of bandwidth constrained multi-hop wireless networks [1]-[5]. Though these solutions provide better capacity figures, their work mainly focus on the assumption of minimum power. Besides these theoretical studies, there are also lots of research efforts, e.g., [7],[8] and the references therein, which focus on developing power-optimal algorithms for maximizing the lifetime of the energy constrained wireless networks such as ad hoc and sensor networks. Contrary to these existing research efforts [2]-[8], this paper looks at the power problem from a different angle and plans to leverage power to enhance the SNR at the receiver. Interestingly, there are few supporting arguments [9]-[11] where the researchers study the power problem by various approaches including optimization, simulations and manifest that for some network configurations, capacity can be indeed maximized by properly increasing the transmission power. There are also research efforts [12]-[15] that investigate the problem of wireless scheduling under physical SINR model [1] by utilizing the power-control. Our work essentially differs from these existing research efforts due to the following aspects: (a) [12]-[15] exploit power-control either to realize an acceptable level of signal quality at each receiver in the presence of several concurrent transmissions or to minimize the time needed to schedule all the wireless transmissions under SINR model. However, our work is geared towards enhancing the SNR at each receiver, by leveraging power, to a value exceeding this acceptable signal quality and then study its effect on the capacity. (b) [9]-[15] in fact addressed the power issue in single-channel wireless networks and in this work we focus on investigating the effect of exploiting power in a MR-MC wireless network.

III. System Model

A. Network Model and Assumptions

Consider a network of n nodes in a torus of unit area. Let X_i, $1 \leq i \leq n$, denote the location of node i. We will use X_i to denote a node as well as its location. Similar to [2], we also assume a slotted model for the convenience of elucidation. Let $\{X_i, X_{R(i)}) : i \in T_X\}$ be the set of all transmitter-receiver pairs in some particular slot, where T_X denote the set of transmitters. We use P_i to denote the transmit power for pair $(X_i, X_{R(i)})$. Let the transmission radius and interference radius be denoted as $r(i)$ and $r_I(i)$. In this paper, we denote all the constants by $k_i,\ i > 0$. We further take the following assumptions: (A.1) There are c channels in the network and each node is equipped with ϕ interfaces (or radios), $1 \leq \phi \leq c$, commonly referred to as (ϕ, c) network. We also assume that all nodes transmit on an ideal channel without channel fading. Further, we denote $\tilde{c}$ as the minimum number of channels with which the network can achieve the maximum set of conflict-free transmissions; (A.2) An interface is capable of transmitting or receiving data on any selected channel and at different slots, the interface can switch the channel. We also assume that channel switching can be immediately implemented without delay; (A.3) Each node $X_i : i \in T_X$ is constrained to a maximum transmit power P_{max} such that $P_{min} \leq P_i \leq P_{max}$, where P_{min} is the minimum transmit power; (A.4) In the MR-MC context, by transmission we imply the communication between a sending and a receiving radio interface; and (A.5) This paper interchangeably uses $\log_2(.)$ and $\log(.)$ to represent logarithm to base 2.

B. Impact of Power on Interference Model

We study the capacity of the proposed solution under the protocol interference model [2]. In this model, the transmission from node $X_i, i \in T_X$, is successfully received by the receiver $X_{R(i)}$ only if the receiving node $X_{R(i)}$ is in the transmission radius of the corresponding transmitting node X_i and is out of the interference radius of all other transmitting nodes $X_k, k \in T_X \setminus i$. In [2], Gupta and Kumar do not explicitly take into account the dependency of power of each node on the interference under the protocol model. Thus, we modify the interference model by considering the transmit power level of each node. To derive the necessary condition for a successful transmission, we first quantify the transmission and interference radius of a node in the wireless network [3].

1) Transmission and Interference Radii: A data transmission from node X_i to its receiver $X_{R(i)}$ is successful only if the received signal strength at $X_{R(i)}$ exceeds a power threshold, say η i.e., $\frac{P_i}{(d_{i,R(i)})^\alpha} \geq \eta$, where $d_{i,R(i)} = \|X_i - X_{R(i)}\|$ is the physical distance between two nodes X_i and $X_{R(i)}$ and $\alpha \geq 2$ is the path loss exponent. Then, the transmission radius of the node, denoted as $r(i)$ is given as $r(i) = (\frac{P_i}{\eta})^{1/\alpha}$. Similarly, we also know that a transmission from node X_i is successfully received at $X_{R(i)}$ only if the interference power level does not exceed a threshold, say β at the receiver where $\beta < \alpha$. Following the same derivation for the transmission radius, the interference radius of a node, denoted as $r_I(i)$ is obtained as $r_I(i) = (\frac{P_i}{\beta})^{1/\alpha}$(refer to [3] for details).

2) Condition for Successful Transmission: For a given channel $m \in [1, c]$, we present the necessary conditions to schedule a successful transmission from node X_i to its receiver node $X_{R(i)}$ under the protocol model.

1. The receiving node $X_{R(i)}$ must be physically within the transmission radius of node X_i i.e.,

$$d_{i,R(i)} \leq r(i) = (\frac{P_i}{\eta})^{1/\alpha} \tag{1}$$

2. The receiving node $X_{R(i)}$ should lie outside the interference radius of any other node $k \in T_X \setminus i$ that is transmitting in the same channel, i.e.,

$$d_{k,R(i)} \geq r_I(k) = (\frac{P_k}{\beta})^{1/\alpha} \tag{2}$$

C. Link Capacity and Power Enhancement

We use Shannon's capacity formula for the additive white Gaussian noise channel to model the data rate. In this model, the data rate is a function of the signal-to-interference-plus-noise ratio (SINR) at the receiver. For instance, the data rate from transmitter X_i to its receiver $X_{R(i)}$ is given in bits/sec by

$$B_i = W_m \log_2(1 + \frac{\frac{P_i}{(d_{i,R(i)})^\alpha}}{N_0 W_m + \sum_{k \in T_X, k \neq i} \frac{P_k}{(d_{k,R(i)})^\alpha}}) \tag{3}$$

where W_m is the bandwidth of the channel $m \in [1, c]$ in hertz, and N_0 is the white noise spectral density in watts/hertz.

In this work, we consider two power modes termed as the **basic** and the **power** mode: (1). In the **basic mode**, the nodes in a MR-MC network communicate at a minimum power level P_{min} and this mode is similar to the one in [6]. Note that we use P_{min} (for convenience) to indicate that every transmitting node operates in the basic mode, but different nodes may have different minimum power values. From equation (1), it turns out that the power level of a node should be at least $\eta d_{i,R(i)}^\alpha$. For the convenience of our capacity analysis, we set the minimum transmit power P_{min} as $(N_0 W_m + k_1)\eta d_{i,R(i)}^\alpha$ and $N_0 W_m \eta d_{i,R(i)}^\alpha$ in the region $c < \frac{n\phi}{2}$ and $c \geq \frac{n\phi}{2}$ respectively. One may notice that such setting will not impact our asymptotic analysis result and $k_1(= \sum_{k \in T_X, k \neq i} \frac{P_k}{(d_{k,R(i)})^\alpha})$ is the sum of the interference at a receiver due to all the simultaneous transmissions, presented by Lemma 2. (2) In the **power mode**, the nodes in a MR-MC network transmit at a power level P_i which is larger than P_{min}, where $P_{min} < P_i \leq P_{max}$. Particularly in this mode, we investigate the capacity of MR-MC wireless networks enhancing power in the following two regimes: (a) $\tilde{c} < c < \frac{n\phi}{2}$, we term this regime as **channel-constraint**, where co-channel interference exists and the power level a node can choose in turn depends on $\tilde{c}$ and c. We set the power level of nodes in this regime as $P_{min}(\frac{c}{\tilde{c}})^{\frac{\alpha}{2}}$; and (b) $c \geq \frac{n\phi}{2}$, we term this regime as **power-constraint**, where each transmission could have a dedicated channel and each transmitter can use the power up to P_{max}. We adopt two maximum power assignments for nodes in this regime, **polynomial** and **exponential**. In the polynomial power mode, each node choose $P_{max} = N_0 W_m \eta d_{i,R(i)}^\alpha n^K = P_{min} n^K$, where the power level polynomially increases with n. In the exponential power mode, each node choose $P_{max} = N_0 W_m \eta d_{i,R(i)}^\alpha 2^{\frac{n\phi}{2}} = P_{min} 2^{\frac{n\phi}{2}}$, where the power level increases exponentially with n. Moreover, in Lemma 2 we show that the accumulated interference at receiver $X_{R(i)}$, under the *basic mode* and the *channel-constraint region*, due to all simultaneous transmissions can be bounded by a constant $k_1 = k_0\beta$. We can, therefore, rewrite the equation in (3) as follows:

$$B_i \approx W_m \log_2(\frac{\frac{P_i}{(d_{i,R(i)})^\alpha}}{N_0 W_m + k_1}) \text{ [for } c < \frac{n\phi}{2}]. \tag{4}$$

Also, as each node obtains the same SNR level under the chosen power assignments, in the sequel we assume that $B_i = B, \ \forall i \in T_X$. Throughout the paper, we denote the data rate

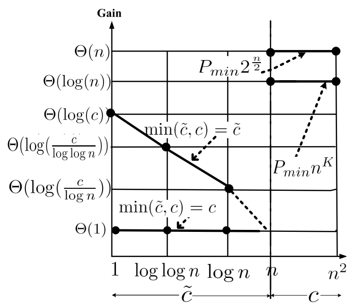

Fig. 2. Gain of *power* mode over *basic* in arbitrary case for $\alpha = \eta = 2, \phi = K = 1$ (Figure not to scale).

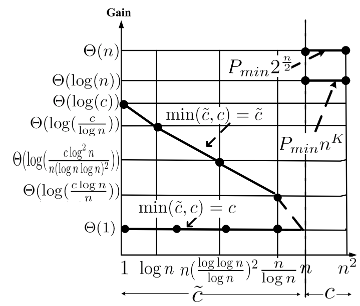

Fig. 3. Gain of *power* mode over *basic* in random case for $\alpha = \eta = 2, \phi = K = 1$ (Figure not to scale).

B of a sender-receiver link in the channel-constraint region of power mode as B_{power}. In the power-constraint region, since each node can communicate at the full power level P_{max} we denote the achievable data rate as B_{Max}. Lastly when nodes choose minimum power level, we denote the data rate achieved in basic mode as B_{basic}.

D. Channel Model

As in [6], we consider the following basic assumptions on the channel model: (a) channel model I: In this model, we have $W_m = \frac{W}{c}$, $\forall m \in [1, c]$. Intuitively, we can see that as the number of channels c increases, the bandwidth for each channel decreases and hence the data rate supported by each channel will be less; *see* eq (4); (b) channel model II: In this model, we have $W_m = W$, $\forall m \in [1, c]$. In contrast to channel model I, we can observe that each channel has a bandwidth of W and hence, higher data rate. The derivation of all the proofs in this work is based on the assumption of channel model I. However, all the results under channel model II can be obtained by replacing W by $W.c$.

IV. Main Results and Discussion

This section summaries our main results of network capacity analysis under both the basic mode and power mode. In the cases where power mode is not available, only results under the basic mode are presented. Besides, we plot the gain of *power* mode over *basic* mode for arbitrary and random placement of nodes in Fig. 2 and 3, respectively. We determine the gain as follows: power mode capacity/basic mode capacity.

Theorem 1. The network capacity of a (ϕ, c) arbitrary network with n nodes is given as:
1. When $\tilde{c} < c < \frac{n\phi}{2}$, the network capacity is $\Theta(B_{power}\sqrt{\frac{n\phi\tilde{c}}{\pi}})$ and $\Theta(B_{basic}\sqrt{\frac{n\phi\tilde{c}}{\pi}})$ respectively for *power* and *basic* modes.
2. When $c \leq \tilde{c} < \frac{n\phi}{2}$, the network capacity for *basic* mode is $\Theta(\frac{W}{\sqrt{c}}\log(\eta)\sqrt{\frac{n\phi}{\pi}})$.
3. When $c \geq \frac{n\phi}{2}$, the network capacity is $\Theta(\frac{B_{Max}n\phi}{2})$ and $\Theta(\frac{B_{basic}n\phi}{2})$ for *power* and *basic* modes. □

Theorem 2. The network capacity of a (ϕ, c) random network with n nodes is given as:

1. When $\tilde{c} < c < n\phi/2$ and $\tilde{c} = O(\log n)$, the network capacity is $\Theta(B_{power}.\tilde{c}\sqrt{\frac{n}{\log n}})$ and $\Theta(B_{basic}.\tilde{c}\sqrt{\frac{n}{\log n}})$ for *power* and *basic* modes respectively. When $\tilde{c} \geq c$ and $c = O(\log n)$, the network capacity is $\Theta(W\log(\eta)\sqrt{\frac{n}{\log n}})$ for *basic* mode.
2. When $\tilde{c} < c < n\phi/2$, $\tilde{c} = \Omega(\log n)$ and also $O(n(\frac{\log\log n}{\log n})^2)$, the network capacity for *power* and *basic* modes are
$\Theta(B_{power}\sqrt{\frac{n\phi\tilde{c}}{\pi}})$ and $\Theta(B_{basic}\sqrt{\frac{n\phi\tilde{c}}{\pi}})$ respectively. When $\tilde{c} \geq c$ and $c = \Omega(\log n)$ and also $O(n(\frac{\log\log n}{\log n})^2)$, the network capacity is $\Theta(\frac{W}{\sqrt{c}}\log_2(\eta)\sqrt{\frac{n\phi}{\pi}})$ for *basic* mode.
3. When $\tilde{c} < c < n\phi/2$, $\tilde{c} = \Omega(n(\frac{\log\log n}{\log n})^2)$, the network capacity is $\Theta(\frac{B_{power}n\phi}{F(n)})$ and $\Theta(\frac{B_{basic}n\phi}{F(n)})$ under *power* and *basic* modes respectively. When $\tilde{c} \geq c$ and $c = \Omega(n(\frac{\log\log n}{\log n})^2)$, the network capacity is $\Theta(\frac{B_{basic}n\phi}{F(n)})$ for *basic* mode, where $F(n) = \Theta\left(\frac{\log n}{\log\log n}\right)$.
4. When $c \geq n\phi/2$, the network capacity is $\Theta(\frac{B_{Max}n\phi}{F(n)})$ for *power* mode.
where $B_{power} = \frac{W}{c}\log_2(\eta(\frac{c}{\tilde{c}})^{\alpha/2})$, $B_{basic} = \frac{W}{c}\log_2(\eta)$, $B_{Max} = \frac{W\log_2(\eta.n^K)}{c}$ and $\frac{Wn\phi}{2c}$ defined by (12)-(14). □

It follows from Theorems 1 and 2 that the capacity of basic mode goes down by a factor of $\Theta(\log(\frac{c}{\tilde{c}})^{\frac{\alpha}{2}})$ in the regime $\tilde{c} < c < n\phi/2$ (see the region $\min(\tilde{c}, c) = \tilde{c}$ in Fig 2 and 3) and by a factor of $\Theta(\log n)/\Theta(n)$ (see the region $c = \Omega(n)$ in Fig 2 and 3) in the regime $c \geq n\phi/2$, on contrary to the power mode. (1) **Channel-Constraint Regime**. In the region $\tilde{c} < c < n\phi/2$, the capacity loss of basic mode is due to the fact that the network has already reached the maximum set of coexisting transmissions with $\tilde{c}$ and hence those remaining $c - \tilde{c}$ channels will be unused. This in turn implies that the area occupied by all those concurrent transmissions over $c - \tilde{c}$ channels will be wasted and if the transmission disks are distributed to all the channels, then only $\frac{\tilde{c}}{c}$ of the whole area will be covered by the transmissions on each channel. Alternatively, each transmission disk can occupy an area $\frac{c}{\tilde{c}}$ times larger; the larger area in fact allows each node to scale up its transmission power from P_{min} to $P_{min}(\frac{c}{\tilde{c}})^{\frac{\alpha}{2}}$ to fully utilize the areas over all c channels for higher capacity. Based on such channel allocation and power level, we can obtain a gain of $\log(\frac{c}{\tilde{c}})^{\frac{\alpha}{2}}$ per node without increasing the number of radios. However, when $\tilde{c} \geq c$ we see that there is no point in increasing the transmit power level from P_{min} as all the channels are needed to be exploited for conflict-free transmissions and consequently end up with the same capacity as in [6]. For instance, in Fig. 2 and 3, we see the gain of power mode as $\Theta(1)$ in the region $\min(\tilde{c}, c) = c$. (2) **Power-Constraint Regime**. In a network of n nodes each with ϕ interfaces, there are total of $n\phi$ interfaces in the network. Since each interface cannot transmit and receive at the same time, the maximum number of pairs of interfaces available for concurrent transmission is no more than $n\phi/2$. Thus in the power-constraint region ($c \geq n\phi/2$), each node can transmit at the maximum power level P_{max} without interfering other coexisting transmissions. In this region, as there is no constraint on channels, we choose two power assignments: (a) *polynomial power mode*. Under this assignment, each node can be realized with a gain of $\Theta(\log(n))$ over the basic mode (see Fig 2 and 3). Now, using the fact that the transmissions between nodes occur at a distance of $O(1/\sqrt{n})$ [2], the minimum transmit power of each node will at most $\frac{1}{(\sqrt{n})^{\alpha}}$ which in turn implies that P_{max} is at most $P_{min}(n)^K = \Theta(n^{K-\frac{\alpha}{2}})$. If we choose $K = \alpha/2$, the maximum power can be bounded by an order of $\Theta(1)$, independent of n. One may note that the parameter K in fact depends on the amount of power resources available at a node. (b) *exponential power mode*. Under this assignment, each node can be realized with a gain of $\Theta(n)$ over the basic mode. Particularly, we observe that when the channels are split into $\frac{n\phi}{2}$ subchannels, the capacity of the arbitrary and random network under this power level are $\Theta(Wn)$ and $\Theta(Wn\frac{\log\log n}{\log n})$ respectively. Alternatively, the throughput available to each node is at the order of $\Theta(W)$ which is a contrasting result to [6] that claims the end-to-end throughput of each node under channel partition approaches zero when $c = \frac{n\phi}{2}$. This significant capacity gain follows from the Shannon-Hartley theorem which states that data rate is a function of SNR and hence, as the SNR increases with the power, we obtain an increased data rate, at the rate of $\Theta(n)$, for each partitioned channel. Though channel-constrained region is a more practical scenario and the focus of this paper, we observe that the feasibility of power-constrained region in turn depends on the power availability of a node, as indicated by this example.

V. Some Useful Lemmas

In this section, we derive some results that is used to obtain the capacity bounds of the power mode.

Lemma 1. Receiver Interference Model: In a wireless network under protocol interference model, let $(X_i, X_{R(i)})$ and $(X_k, X_{R(k)})$ be two simultaneous active transmitter-receiver pairs over the same channel, then a disk of radius $\frac{\Delta}{2}(d_{i,R(i)})$ centered at $X_{R(i)}$ and a disk of radius $\frac{\Delta}{2}(d_{k,R(k)})$ centered at $X_{R(k)}$, where $\Delta = (\frac{\sigma\eta}{\beta})^{\frac{1}{\alpha}} - 1$, must be disjoint.

Proof: Let $(X_i, X_{R(i)})$ and $(X_k, X_{R(k)})$ be two active transmitter-receiver pairs. From equation 1, we can compute P_i and P_k as $\geq \eta d^{\alpha}_{i,R(i)}$ and $\geq \eta d^{\alpha}_{k,R(k)}$ respectively. Thus, the preceding equations can be rewritten as follows:

$$P_i = \sigma\eta d^{\alpha}_{i,R(i)} \text{ and } P_k = \sigma\eta d^{\alpha}_{k,R(k)} \tag{5}$$

where $\sigma > 0$. Recall that the protocol model places the following constraints on the relative locations of these nodes and using equation 2, we get:

$$d_{k,R(i)} \geq (\frac{\sigma\eta}{\beta})^{\frac{1}{\alpha}} d_{k,R(k)} \tag{6}$$

$$d_{i,R(k)} \geq (\frac{\sigma\eta}{\beta})^{\frac{1}{\alpha}} d_{i,R(i)} \tag{7}$$

Using the triangle inequality first, we can derive the following relation between the location of the node pairs $(X_i, X_{R(i)})$ and $(X_k, X_{R(k)})$.

$$d_{R(i),R(k)} + d_{k,R(k)} \geq d_{R(i),k}$$

$$d_{R(i),R(k)} \geq (\frac{\sigma\eta}{\beta})^{\frac{1}{\alpha}} d_{k,R(k)} - d_{k,R(k)} \tag{8}$$

Similarly we can write,

$$d_{R(i),R(k)} \geq (\frac{\sigma\eta}{\beta})^{\frac{1}{\alpha}} d_{i,R(i)} - d_{i,R(i)} \tag{9}$$

Adding the inequalities in (8) and (9), we obtain $d_{R(i),R(k)} \geq \frac{\Delta}{2}(d_{i,R(i)} + d_{k,R(k)})$, where $\Delta = (\frac{\sigma\eta}{\beta})^{\frac{1}{\alpha}} - 1$. Note that $X_{R(i)}$ and $X_{R(k)}$ are the receivers and we can deduce this inequality to say that for transmitter-receiver pairs $(X_i, X_{R(i)})$ and $(X_k, X_{R(k)})$ to be active, then a disk of radius $\frac{(\frac{\sigma\eta}{\beta})^{\frac{1}{\alpha}}-1}{2}(d_{i,R(i)})$ centered at $X_{R(i)}$ and a disk of radius $\frac{(\frac{\sigma\eta}{\beta})^{\frac{1}{\alpha}}-1}{2}(d_{k,R(k)})$ centered at $X_{R(k)}$ should not overlap. ■

Remark 1. It follows from Lemma 1 that the radius of the interference disk centered around each receiver directly depends on Δ, where $\Delta = (\frac{\sigma\eta}{\beta})^{\frac{1}{\alpha}} - 1$. For instance, consider two power levels P_{min} and $P_{min}\frac{c}{\tilde{c}}^{\frac{\alpha}{2}}$. From subsection III-C, it follows that $P_{min} = (N_0W_m + k_1)\eta d^{\alpha}_{i,R(i)}$ and hence $P_{min}\frac{c}{\tilde{c}}^{\frac{\alpha}{2}} = (N_0W_m + k_1)\eta d^{\alpha}_{i,R(i)} \left(\frac{c}{\tilde{c}}^{\frac{\alpha}{2}}\right)$. From Eq.5, it also turns out that $P_i = \sigma\eta d^{\alpha}_{i,R(i)}$, $\forall i \in T_X$. This implies that $\sigma > 0$ takes values $(N_0W_m + k_1)$ and $(N_0W_m + k_1)\frac{c}{\tilde{c}}^{\frac{\alpha}{2}}$ respectively for transmit power levels P_{min} and $P_{min}\frac{c}{\tilde{c}}^{\frac{\alpha}{2}}$, where $k_1 = k_0\beta$ is a constant that follows from Lemma 2. Since the radius of the interference disk directly depends on $\Delta = (\frac{\sigma\eta}{\beta})^{\frac{1}{\alpha}} - 1$, one may note that an increase in transmit power can in fact lead to an increase in the radius of the interference disk. In the following lemma, we further show that when each node transmit on a power level, say P ($P_{min} \leq P < P_{max}$) and Δ is in turn tuned to reflect the transmit power level of the node, the sum of interference at a receiver due to all simultaneous transmissions can be bounded by a constant.

Lemma 2. Let (X_a, X_b) be an active transmitter-receiver pair. When $c < \frac{n\phi}{2}$, the accumulated interference at receiver X_b due to all the simultaneous transmitter-receiver pairs under basic mode and channel-constraint regime is bounded by a constant $k_1 = k_0\beta$, where $k_0 = \sum_{y=0}^{\infty} 8\left(\frac{y+1}{(2y+1)^{\alpha}}\right)$.

Proof: We suitably modify the approach in [1] (chapter 9, pp.315) to prove this Lemma. Suppose a network in which each node is transmitting with power level P, $P_{min} \leq P < P_{max}$. Consider a receiver X_b and its transmitter X_a placed at a distance of at most $r(n)$ apart. Let $\delta = (\frac{\sigma\eta}{\beta})^{\frac{1}{\alpha}} r(n)$ be a guard zone around the receiver in which no other transmitters are located (note that the value of δ follows from equation (7) which states the minimum distance between a receiver and other simultaneous transmitters for a successful reception of data). To bound the sum of interference at the receiver X_b, we should take into account all the simultaneous transmissions. Therefore, consider a closely packed disks of radius δ of all simultaneous transmitter-receiver pairs placed around receiver X_b. According to protocol interference model, we know that each of these simultaneous transmitters should be placed at a distance of at least $(2y+1)\delta$ for $y = 0, 1\cdots$ from the receiver. Now consider the annulus between y^{th} and $(y+1)^{th}$ distance from receiver X_b. The number of transmitters inside this annulus cannot be more than $\frac{\pi((2y+3))\delta^2 - \pi((2y+1))\delta^2}{\pi\delta^2}$= $8(y+1)$ transmitters. Thus, an upper bound on the sum of interference power received at X_b can be determined as follows: $\sum_{y=0}^{\infty} \frac{P(8y+1)}{((2y+1)\delta)^\alpha} > \frac{8P}{\delta^\alpha} \sum_{y=0}^{\infty} \frac{y+1}{(2y+1)^\alpha} = \frac{k_0 P}{\delta^\alpha} \leq k_0\beta$, where $k_0 = \sum_{y=0}^{\infty} 8\left(\frac{y+1}{(2y+1)^\alpha}\right)$. The sum converges and is smaller than $8 + \frac{8}{2^\alpha}[\zeta(\alpha-1)]$, where $\zeta(\alpha-1)$ is the Reimann Zeta Function. Moreover, the last inequality stems from the fact $\delta = (\frac{\sigma\eta}{\beta})^{\frac{1}{\alpha}} r(n)$ and $\frac{P}{r(n)^\alpha} = \sigma\eta$; from Remark 1, we know that σ and P is dependent on each other. ■

Remark 2. Lemma 2 holds true only for the basic mode and the channel-constraint regime of the power mode in the region $c < n\phi/2$. Otherwise as $c \geq n\phi/2$, the sum of interference at a receiver can be regarded as negligible, i.e., $k_1 \approx 0$. This is because in a network with n nodes each with half-duplex ϕ interfaces, the number of simultaneous transmissions possible at a given time is no more than $n\phi/2$. Therefore when $c \geq n\phi/2$ channels are available, it implies each transmission on an interface could have a dedicated channel.

Lemma 3. For a random network with n nodes and the cell area of $s(n) = \frac{k_2 \log n}{n}$, $k_2 > 1$ is a constant, there is no empty cell *whp* when $n \to \infty$.

Proof: Let $y = 1/s(n)$ be the number of cells in a torus of unit area and then the probability that each node can be in any cell is given by $1/y = \frac{k_2 \log n}{n}$. Let E_i be the event that cell i is empty. Then, the probability that cell i is an empty cell is $P(E_i) = (1 - \frac{k_2 \log n}{n})^n$. Therefore using union bound $[P(\bigcup_{i=1}^{y} E_i) \leq \sum_{i=1}^{y} P(E_i)]$, we get the probability that an empty cell exists as no more than $\frac{n}{k_2 \log n}(1 - \frac{k_2 \log n}{n})^n$. When $n \to \infty$, this probability is $\lim_{n\to\infty} \frac{n}{k_2 \log n}(1 - \frac{k_2 \log n}{n})^n = \lim_{n\to\infty} \frac{n}{k_2 \log n} e^{-k_2 \log n} = \lim_{n\to\infty} \frac{n^{1-k_2}}{k_2 \log n} = 0$, where the first equality follows from $\lim_{x\to\infty}(1 - \frac{1}{x})^{-x}$=e. ■

Lemma 4. For a (ϕ, c) arbitrary network with n nodes and $c < \frac{n\phi}{2}$, the network capacity is upper bounded by: (a) $O\left(\frac{W}{c}\log(\eta(\frac{c}{\tilde{c}})^{\alpha/2})\sqrt{\frac{n\phi\tilde{c}}{\pi}}\right)$ when $\tilde{c} < c$ for *power* mode; (b) $O\left(\frac{W}{c}\log(\eta)\sqrt{\frac{n\phi\tilde{c}}{\pi}}\right)$ when $\tilde{c} < c$ for *basic* mode; and (c) $O\left(\frac{W}{\sqrt{c}}\log(\eta)\sqrt{\frac{n\phi}{\pi}}\right)$ when $\tilde{c} \geq c$ for *basic* mode.

Proof: We study the transport capacityConsider now the set of all the transmitter-receiver pairs $(X_i, X_{R(i)}) : i \in T_X$ and denote this set by S. We assume that the network operates in a time slotted manner and that the nodes are synchronized. We will also assume that in each slot B bits are transmitted. Then the transport capacity of the network is given by $B\sum_S d_{i,R(i)}$ bit-meters in this slot. From Lemma 1, we know that the disks of radius $\frac{\Delta}{2} d_{i,R(i)}$ centered at each receiver $X_{R(i)}$, will be disjoint. Since the disjoint disks are to be non overlapping, the sum of the areas of the disks on each channel is bounded above by the area of the domain ($1m^2$); thus summing over all the $\min(\tilde{c}, c)$ channels, we have the constraint given by eq.(10). **Remark 3.** In [6], authors assume that c channels are present in the network and hence the summation of the areas of each disk on each channel sum to 1. However, this reasoning is not accurate. Recall from Section 1, when $\tilde{c} < c$ only $\frac{\tilde{c}}{c}$ channels are utilized due to the minimum power assumption and hence, the summation of area of each disk on each channel is upper bounded by $O(\frac{\tilde{c}}{c})$. This also implies that if each node increases its disk radius by $\sqrt{\frac{c}{\tilde{c}}}$, then all existing c channels can be exploited. On the other hand, note that the reasoning of [6] holds true when $\tilde{c} \geq c$. Combining the two conditions $\sum_S \frac{\pi\Delta^2 d_{i,R(i)}^2}{4} \leq \min(\tilde{c}, c)$ and $\sum_S \frac{\pi(\sqrt{\frac{c}{\tilde{c}}}\Delta)^2 d_{i,R(i)}^2}{4} \leq c$ for basic and power mode respectively, we can write:

$$\sum_S d_{i,R(i)}^2 \leq \frac{4\min(\tilde{c}, c)}{\pi\Delta^2} \tag{10}$$

Since there are n nodes with ϕ interfaces in the network, it is not difficult to see that $|S| \leq \frac{n\phi}{2}$. From the Cauchy-Schwarz inequality, we can write

$$\sum_S d_{i,R(i)} \leq \sqrt{\sum_S d_{i,R(i)}^2 \sum_S 1} \leq \sqrt{\frac{n\phi}{2}\sum_S d_{i,R(i)}^2} \leq \sqrt{\frac{n\phi}{2} * \frac{4\min(\tilde{c}, c)}{\pi\Delta^2}} = \frac{1}{\Delta}\sqrt{\frac{2n\phi\min(\tilde{c}, c)}{\pi}} \tag{11}$$

We look at the following cases to determine the data rate B. (1) B_{power}. when $\tilde{c} < c < \frac{n\phi}{2}$, although c channels are present in the network, no more than $\tilde{c}$ channels are utilized under minimum power P_{min}. Hence to completely exploit the existing c channels, each node in the power mode, can increase the disk size by $\sqrt{\frac{c}{\tilde{c}}}$. This in turn implies that each node can scale its power from P_{min} to $P_{min}(\sqrt{\frac{c}{\tilde{c}}})^\alpha$. Substituting $P_{min}(\sqrt{\frac{c}{\tilde{c}}})^\alpha$ in equation (4) and using $W_m = W/c$, we get

$$B_{power} = \frac{W}{c}\log_2\left(\frac{\frac{P_{min}(\sqrt{\frac{c}{\tilde{c}}})^\alpha}{(d_{i,R(i)})^\alpha}}{N_0 W_m + k_1}\right) = \frac{W}{c}\log_2(\eta(\frac{c}{\tilde{c}})^{\alpha/2}) \tag{12}$$

Substituting B_{power} and eq.11 in $B\sum_S d_{i,R(i)}$, we obtain the transport capacity as $\frac{W}{c\Delta}\log_2(\eta(\frac{c}{\tilde{c}})^{\alpha/2})\sqrt{\frac{2n\phi\tilde{c}}{\pi}}$. (2) B_{basic}.

when $\tilde{c} < c < \frac{n\phi}{2}$, each node sends at the minimum power level P_{min} in the basic mode, the data rate B_{basic} is given by:

$$B_{basic} = \frac{W}{c}\log_2(\eta) \quad (13)$$

Substituting B_{basic} and eq.11 in $B\sum_{S} d_{i,R(i)}$, we obtain the transport capacity as $\frac{W}{c\Delta}\log_2(\eta)\sqrt{\frac{2n\phi\tilde{c}}{\pi}}$.

(3) B_{basic}. when $\tilde{c} \geq c < \frac{n\phi}{2}$, each node can communicate only at the minimum power level P_{min} as all the existing c channels are needed to be exploited for conflict-free transmissions. Thus, substituting $B_{basic} = \frac{W}{c}\log_2(\eta)$ and eq.11 in $B\sum_{S} d_{i,R(i)}$, we obtain the transport capacity as $\frac{W}{\sqrt{c}\Delta}\log_2(\eta)\sqrt{\frac{2n\phi}{\pi}}$. ■

Lemma 5. For a (ϕ, c) arbitrary network with n nodes and $c \geq \frac{n\phi}{2}$, the network capacity is $O(\frac{B_{Max}n\phi}{2})$ and $O(\frac{B_{basic}n\phi}{2})$ for power and basic modes respectively, where B_{basic} and B_{Max} are defined in eq. (13)-(14).

Proof: In a network of n nodes each with ϕ radios, the maximum number of simultaneous transmissions feasible is $\frac{n\phi}{2}$. Since $c \geq \frac{n\phi}{2}$, each communicating pair can transmit at the maximum power level P_{max} (P_{min}) to obtain a data rate of B_{Max} (B_{basic}) under power (basic) mode. Further, the maximum distance a bit can travel in the network is $O(1)$ meters. Thus, the network capacity is at most $O(\frac{B_{Max}n\phi}{2})$ and $O(\frac{B_{basic}n\phi}{2})$ bit-meter/sec for power and basic modes respectively. ■

VI. Capacity Analysis

A. Random Networks

In this section, we characterize the upper and lower bounds of the power mode in random networks. We assume that n nodes are randomly deployed on the surface of a torus of unit area. Each node selects a destination randomly to which it transmits $\lambda(n)$ bits/sec. The maximum value of $\lambda(n)$ that can be supported by every source-destination pair with high probability (*whp*) is defined as the per-node throughput of the network [2], [6]. Since there are total of n flows—the traffic from a source node to destination node is termed as a flow)—the network capacity is defined to be $n\lambda(n)$. In the sequel, we denote $d(n)$ as the average distance between two nodes in a random network with n nodes.

1) An Upper Bound on Capacity: The capacity of MR-MC networks is limited by the following three constraints [6], (a) connectivity, (b) interference and (c) destination bottleneck-power constraint and each of them is used to obtain a bound on the network capacity in random settings for given parameters $\tilde{c}$ and c. In particular when $\min(\tilde{c}, c) < n\phi/2$, the minimum of the three bounds is an upper bound on the network capacity. And, when $c \geq n\phi/2$, the upper bound on capacity is given by the destination bottleneck-power constraint.

Connectivity Constraint: In random networks, this constraint is shown as the necessary condition, See [2], to ensure that the network is connected *whp*. It follows from [2] that the number of concurrent transmissions on any particular channel is no more than $\frac{1}{\pi\frac{\Delta}{2}^2 d(n)^2}$. Observing that each transmission over the mth channel is of B bps (See section III-C) by summing all the transmissions taking place at the same time over all the $\min(\tilde{c}, c)$ channels, we obtain $\frac{4}{\pi\Delta^2 d(n)^2}\sum_{m=1}^{\min(\tilde{c},c)} B = \frac{4B\min(\tilde{c},c)}{\pi\Delta^2 d(n)^2}$ bits/second. Moreover, since each source-destination of a flow is separated by an average of $\Theta(1)$—due to the assumption of a torus of unit area—distance, we have the average number of hops as $\Theta(\frac{1}{d(n)})$ between each source-destination pair. Now, there are total of n sources and when each source generates $\lambda(n)$ bits/second, then the total number of bits per second served by the entire network is at least $n\lambda(n)/d(n)$. To guarantee that all the required traffic is carried, we thus need $\frac{n\lambda(n)}{d(n)} \leq \frac{4B\min(\tilde{c},c)}{\pi\Delta^2 d(n)^2}$. In a precursor result [2], we see that $d(n) > \sqrt{\frac{\log n}{\pi n}}$ is necessary to ensure connectivity in random networks *whp*, and hence the distance between two nodes should be at least $k_3\sqrt{\frac{\log n}{\pi n}}$, where $k_3 > 1$. This in turn implies that minimum transmit power P_{min} should be $(N_0 W_m + k_1)\eta(\frac{k_3^2\log n}{\pi n})^{\alpha/2}$. Now, substituting $d(n) > \sqrt{\frac{\log n}{\pi n}}$ we have $n\lambda(n) \leq \frac{4B\sqrt{n}\min(\tilde{c},c)}{\Delta^2\sqrt{\pi\log n}}$. Following the same derivation techniques in eq.(12-13), we obtain the final bounds on capacity as follows: (a) $O\left(B_{power}.\tilde{c}\sqrt{\frac{n}{\log n}}\right)$ when $\tilde{c} < c$ under *power* mode; (b) $O\left(B_{basic}.\tilde{c}\sqrt{\frac{n}{\log n}}\right)$ when $\tilde{c} < c$ under *basic* mode; and (c) $O\left(W\log_2(\eta)\sqrt{\frac{n}{\log n}}\right)$ when $\tilde{c} \geq c$ under *basic* mode.

Interference Constraint: The capacity of random networks using multiple channels is also constrained by interference. In arbitrary setting, we capture the capacity of the power mode in the region $\tilde{c} < c < n\phi/2$, See Lemma 4, as $O\left(B_{power}\sqrt{\frac{n\phi\tilde{c}}{\pi}}\right)$ bit-meter/second. Since this upper bound is optimal for all situations, it applies to random networks as well. Also, in a random network each of the n source-destination pairs are separated by an average of $\Theta(1)$ meters and thus we have the capacity of power mode in random setting as at most $O\left(B_{power}\sqrt{\frac{n\phi\tilde{c}}{\pi}}\right)$ bits/sec.

Destination Bottleneck-Power Constraint: In a random network, each source randomly selects a destination and as a result, the network capacity will also be restricted by the number of flows towards a destination node, D. The maximum number of flows, $F(n)$, from source nodes to a chosen destination is bounded by $\Theta\left(\frac{\log n}{\log\log n}\right)$ [6], which in turn leads to a network capacity under this constraint as $\frac{Bn\phi}{F(n)}$ bits/second. We now have the following two cases: (i) $\tilde{c} < c < n\phi/2$. In this regime, each node can transmit at power $P_{min}(\sqrt{\frac{c}{\tilde{c}}})^{\alpha}$ to utilize all the existing c channels for power mode, and thus following the same approach in eq.(12), the maximum network capacity under the destination bottleneck constraint is given by $O(\frac{B_{power}n\phi}{F(n)})$. However in basic mode, since each node can transmit only at the minimum power level P_{min}, we obtain the capacity as $O(\frac{B_{basic}n\phi}{F(n)})$. (ii) $c \geq n\phi/2$. In this regime, it implies that each node can tune its interface to a different channel and can transmit at the polynomial or exponential maximum power level, without interfering other transmissions in the network. Therefore using P_{max}, $W_m = W/c$ in equation (4) and setting $k_1 \approx 0$ (follows from Lemma 2), we obtain B_{Max} as

$$B_{Max} = \begin{cases} \frac{W \log_2(\eta . n^K)}{c} & \text{polynomial;} \\ \frac{W(\log_2 \eta + \frac{n\phi}{2})}{c} \approx \frac{Wn\phi}{2c} & \text{exponential;} \end{cases} \quad (14)$$

and the maximum network capacity for destination bottleneck-power constraint as $O(\frac{B_{Max} n\phi}{F(n)})$.

2) A Lower Bound on Capacity: In this section, we provide the lower bound construction for a single interface multi-channel network according to Lemma 2 from [6] which states that the capacity for the (ϕ, c) network can be obtained by replacing c in the results with $\frac{c}{\phi}$.

To prove that the upper bound in Section VI-A1 can be quite tight, we construct a network and then design both a routing scheme and a transmission schedule. Our routing scheme follows a cell-based approach. We divide the unit torus into equal-sized squares (or cells) of area

$$s(n) = \min \left(\max\left(\frac{\log n}{n}, \frac{\min(\tilde{c}, c)}{n}\right), \left(\frac{\log \log n}{\log n}\right)^2 \right) \text{ and set}$$

the transmission distance as $d(n) = \sqrt{8s(n)}$ so that a node in one cell can transmit to some other node in its four neighboring cells. Specifically the size of each cell is chosen to satisfy the three constraints given in Section VI-A1. Intuitively, the area of the cell in fact determines the range of transmission, $\Theta(\sqrt{s(n)})$, and correspondingly the minimum number of channels, $\tilde{c}$, for reaching conflict-free in a neighborhood. For instance, it follows from [1] that $\tilde{c} \approx n\Delta^2 d(n)^2$; implying that $\tilde{c}$ should be at least $ns(n)$ in this construction. Hence for $s(n) = \frac{50 \log n}{n}$, we see that $\tilde{c}$ is at least $O(\log n)$. Our next step is to draw a line to connect each source-destination (S-D) pair which passes through some cells. One node is chosen from each of these cells to relay the traffic from the source node to its destination. Such a routing scheme requires at least one node in each cell. From Lemma 3, it follows that the probability of existence of an empty cell is low if $s(n) = \frac{50 \log n}{n}$. We next bound the number of nodes that are present in each cell of size $s(n)$. We have the following lemma from [4].

Lemma 6. (Ref.[4]) If $s(n)$ is greater than $\frac{50 \log n}{n}$, each cell has $\Theta(ns(n))$ nodes per cell, whp. □

Once the number of nodes in a cell is determined, our next step is to bound the number of cells that will interfere with a given cell, which in turn is given by the following Lemma from [4].

Lemma 7. (Ref.[4])The number of cells that interfere with any given cell is bounded by a constant $k_4 = 16(2 + \Delta)^2$, i.e, independent of n and s(n).

Routing Scheme: In the cell based routing scheme we choose a route with the shortest distance to forward packets. A straight line, S-D, is passing through the cells where nodes S and D are located [here, S refers to the source of the flow and D refers to the final destination of the flow]. Packets are delivered along the cells lying on the S-D line. Then, we choose a node within each cell lying on the straight line to carry that flow. For load balancing [6], we assign each flow to a node within a cell that has been assigned the least number of flows. Thus, each node has nearly the same number of flows. We use the result in [4] to bound the number of S-D lines passing through any cell. We state their lemma here.

Lemma 8. (Ref.[4]) The maximum number of lines passing through any cell is $O(n\sqrt{s(n)})$ whp. □

It follows from Lemma 6 that each cell has $\Theta(ns(n))$ nodes with *whp*. Besides, each cell has $O(n\sqrt{s(n)})$ flows based on Lemma 8 and hence each node in the network is assigned at most $O(\frac{1}{\sqrt{s(n)}})$ flows due to load balancing. Noting that each node in the cell is simultaneously a source S, a potential destination D and a relay for other S-D pairs, the total flows assigned to every node is $O(1 + F(n) + \frac{1}{\sqrt{s(n)}}) \approx O(\frac{1}{\sqrt{s(n)}})$.

Scheduling Scheme: Though we construct the scheduling scheme for single interface multi-channel network, these results can be easily extended to multi-interface network by using the lemma from [6]. Now, any transmissions in this model must satisfy the following two constraints: (a) each interface only allows one transmission/reception at the same time; and (b) any two transmissions on any channel should not interfere with each other. We propose a time-division multi-access (TDMA) scheme to schedule transmissions [2], [6], which satisfy the aforementioned constraints. In this scheme, a second is divided into a number of slots and at most one transmission/reception is scheduled at every node during slot which satisfies the constraint (a). Further, each slot is divided into mini-slots and in each mini-slot, each transmission satisfies the constraint (b). First, noting that the total flows assigned to any node is $O(\frac{1}{\sqrt{s(n)}})$ and each interface allows only one transmission/reception at the same time, we divide every one second time period into $O(\frac{1}{\sqrt{s(n)}})$ slots. Thus, each slot has a length of $\frac{1}{1/\sqrt{s(n)}} = \Omega(\sqrt{s(n)})$ seconds. Second, we divide each slot into mini-slots to satisfy constraint (b). We build a schedule that assigns a transmission to a node in a mini-slot within a slot over a channel. We construct a conflict graph in which nodes represent the vertices of the graph and edges denotes interference between two nodes. Based on Lemma 7, every cell has at most k_4 interfering cells and each cell has $\Theta(ns(n))$ nodes based on Lemma 6. Hence, each node has at most $O(k_4 ns(n))$ edges in the conflict graph. If we use different vertex-color to represent each time slot, then the scheduling problem reduces to the well-studied vertex-color problem. Hence the required number of colors is at most $1 + k_4 ns(n) \approx k_5 ns(n)$, where k_5 is a constant. We now schedule the interfering nodes either on different channels or on different minislots on the same channel. The following channel allocation policy is employed: (a) $min(c, \tilde{c}) = \tilde{c}$: In basic mode, $\tilde{c}$ channels are necessary to reach conflict-free state and so all those interfering nodes will be allocated among $\tilde{c}$ channels. However in power mode, each node will leverage a power level of $P_{min}(\sqrt{\frac{c}{\tilde{c}}})^{\alpha}$ to utilize all the existing c channels. Thus, we will allocate those interfering neighbors among c existing channels. (b) $min(c, \tilde{c}) = c$: In this case, we will allocate all the interfering nodes among c channels. As a result, we can divide each slot into $\left\lceil \frac{k_5 n s(n)}{\min(\tilde{c},c)} \right\rceil$ mini-slots on every channel and assign the mini-slots on each channel from 1 to $\left\lceil \frac{k_5 n s(n)}{\min(\tilde{c},c)} \right\rceil$. Now, we analyze the achievable throughput, $\lambda(n)$, of this network. Recall that each slot has a length of $\Omega(\sqrt{s(n)})$ seconds and each slot is further divided into $\left\lceil \frac{k_5 n s(n)}{\min(\tilde{c},c)} \right\rceil$ mini-slots over every channel. Therefore, each mini-slot has a length of $\Omega(\frac{\sqrt{s(n)}}{\left\lceil \frac{k_5 n s(n)}{\min(\tilde{c},c)} \right\rceil})$. Since each channel can transmit at the rate of B bps, in each minislot $\lambda(n) = \Omega(\frac{B\sqrt{s(n)}}{\left\lceil \frac{k_5 n s(n)}{\min(\tilde{c},c)} \right\rceil})$ can be transported. Noting that $\left\lceil \frac{k_5 n s(n)}{\min(\tilde{c},c)} \right\rceil \leq \frac{k_5 n s(n)}{\min(\tilde{c},c)} + 1$, we have $\Omega(\frac{B\sqrt{s(n)}\min(\tilde{c},c)}{k_5 n s(n) + \min(\tilde{c},c)})$. Note that we choose $d(n) = \sqrt{8s(n)}$ as the maximum transmission distance with which a node can transmit to some node in its four neighboring cells; This in turn implies that the minimum transmit power should be at most $\sigma\eta d(n)^{\alpha}$. Also since the maximum distance is of $\sqrt{8s(n)}$, we have the distance between any two neighboring nodes as $O(\sqrt{s(n)})$. We now have the following lower bounds on network capacity: (a) When $\tilde{c} < c < n/2$ and $c \geq n/2$, the network capacity in basic mode is $\Omega(\min(\frac{B_{basic}.\tilde{c}}{\sqrt{s(n)}}, B_{basic} n\sqrt{s(n)}))$ (b) When $\tilde{c} < c < n/2$, the network capacity in power mode is $\Omega(\min(\frac{B_{power}.\tilde{c}}{\sqrt{s(n)}}, B_{power} n\sqrt{s(n)}))$ respectively. (c) When $c \leq \tilde{c} < n\phi/2$ and $c \geq n/2$, the network capacity in basic mode is $\Omega(\min(\frac{B_{basic}.c}{\sqrt{s(n)}}, B_{basic} n\sqrt{s(n)}))$.

Substituting for $s(n)$ $=\min(\max(\frac{\log n}{n}, \frac{\min(\tilde{c},c)}{n}), \frac{1}{F(n)^2})$, we have the capacity bounds given by Theorem 2. Moreover, when $c \geq n/2$ each node can utilize the maximum power P_{max} to obtain the network capacity for *power* mode as $\Omega(\min(\frac{B_{Max}.\tilde{c}}{\sqrt{s(n)}}, B_{Max} n\sqrt{s(n)}))$.

B. Arbitrary Networks

In Lemma 4 and 5, we captured the upper bounds on arbitrary networks. Similar to random networks, based on

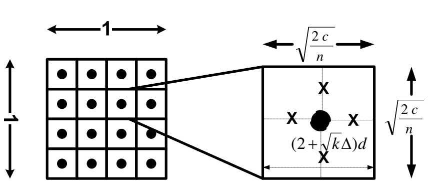

Fig. 4. Figure on the left shows the arrangement of the transmitters in the domain of area = $1m^2$. The distance between each transmitter is $(2 + \sqrt{k}\Delta)d$, where $k = \frac{c}{\min(\tilde{c},c)}$, and the receiver of the corresponding transmitter can be placed at any of the locations marked as X, which is at a distance of d from the transmitter. From this arrangement, the distance between two receivers is $(\sqrt{k}\Delta)d$ and hence, according to Lemma 1, the disks of radius $(\sqrt{k}\Delta)\frac{d}{2}$ centered around each receiver do not overlap.

Lemma 2 from [6], we provide the lower bound construction for single radio multi-channel network. The proof is sketched here for brevity. Consider a torus of unit area. We divide the domain into $\frac{n}{2c}$ cells and place c transmitter-receiver pairs on each cell. Consider the region $c < \frac{n}{2}$. From Fig. 4, it can be verified that there are total of $\frac{n}{2c} * c = \frac{n}{2}$ coexisting transmissions and each transmitting at a rate of B_{power} over a distance of $d = \frac{\sqrt{\frac{2c}{n}}}{2+\sqrt{k}\Delta}$, where $k = \frac{c}{\min(\tilde{c},c)}$. Hence the total capacity of the network is $\frac{B_{power} nd}{2}$ bit-meters/sec. Substituting for k and d, we obtain the network capacity as $\Omega(\frac{W\sqrt{n\tilde{c}}}{c}\log_2(\eta(\frac{c}{\tilde{c}})^{\alpha/2}))$ for power mode. Now consider $c \geq \frac{n\phi}{2}$. In this case, we partition the area into $\frac{n}{2g}$ cells, where $g = \min(c, \frac{n}{2})$, and place g communicating pairs in each cell. Each pair communicate at a rate of B_{Max} over a distance of $d = \sqrt{\frac{2g}{n}} = 1$ (as $g = \min(c, \frac{n}{2})$ and $c \geq \frac{n\phi}{2}$). From such a construction, it can be verified that there are total of $\frac{n}{2g} * g = \frac{n}{2}$ coexisting transmissions and the total capacity of the network as $\frac{B_{Max} n}{2}$ bit-meters/sec.

VII. Conclusion

The findings of this paper mainly stipulate that power is a crucial factor in MR-MC networks and hence, by intelligently leveraging it one may be possible to realize significant gain on capacity of MR-MC wireless networks. The results derived in this work particularly reveal that, when there are sufficient number of channels $c < \frac{n\phi}{2}$ or $c \geq \frac{n\phi}{2}$, then exploiting co-channel enlarging effect with power can help to realize higher capacity.

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