Extracting More Capacity from Multi-Channel Multi-Radio Wireless Networks by Exploiting Power

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Abstract—**Transmission power plays a crucial role in the design and performance of wireless networks. The issue is therefore complex since an increase in transmission power implies that a high quality signal is received at the receiver and hence an increase in channel capacity. Conversely, due to the shared nature of the wireless medium an increase in transmission power also implies high interference in the surrounding region and hence a quadratic reduction in the capacity of wireless networks. Recent literatures indicate that employing multiple channels can mitigate the negative effects of wireless interference and thus greatly improve the overall network capacity. Therefore, it is worth investigating the effect of exploiting power on the capacity of multi-channel multi-radio (MC-MR) wireless networks. Specifically, in this paper we address the following questions: (a) Can we maximize the capacity of MC-MR wireless networks by exploiting power? (b) Under what criteria can we increase the transmission power of the nodes in a MC-MR network?**

When n nodes each with m half-duplex interfaces are optimally deployed in a torus of unit area, traffic patterns are optimally assigned, each transmission's range is optimally chosen and in the presence of c channels, we show that in contrast to the setting where nodes transmit at minimum power level P_0 the transport capacity, measured in bit-meters per second, of MC-MR network exploiting power is increased by $\Theta(\frac{c}{c_{min}})$ in region $c_{min} < c < mn/2$ and by $\Theta(n)$ in region $c \geq mn/2$ when nodes tune to transmit power level of $P_0(\frac{c}{c_{min}})^{\frac{\alpha}{2}}$ and $P_0 n^{\frac{\alpha}{2}}$ respectively—where c_{min} is the minimum number of channels required to achieve conflict-free transmissions in a network. Our analysis also sheds light into several insights that designers may want to consider to improve the performance of energy-efficient bandwidth-constrained wireless networks.

I. INTRODUCTION

In their seminal paper [1]-[2], Gupta and Kumar first derived the capacity of ad hoc wireless networks in the limit as the number of nodes, n, grows to an arbitrarily large level. Under this model, the authors indicate that regardless of the initial location of the nodes and traffic pattern in a fixed area, the throughput in bit-meters/second cannot grow faster than $\sqrt{n}$. Consequently, the end-to-end throughput available for each node is $O(1/\sqrt{n})$ which in turn approaches zero as the number of nodes n increases. This pessimistic result stems from the fact that most communication has to occur between nearest neighbors, at distances of $O(\frac{1}{\sqrt{n}})$, with each packet going through many other nodes (serving as relays) before

reaching the destination. Therefore, it might appear that by reducing the number of hops traversed by each packet (i.e., by increasing the transmission power i.e., range) and thus the burden on nodes serving as relays, the throughput available to each node can be improved. However, the authors [2] point out that increasing transmission range (or power) may reduce the capacity due to spatial concurrency constraints and hence, one may really need to reduce the transmission power (or range) to as small a value as possible without sacrificing the connectivity.

Subsequently, on the assumption of minimal transmission power, a plethora of literatures were proposed to improve the capacity bounds of wireless networks by employing various techniques ranging from mobility [4], [5], [6] to multi-channel multi-radio [7], UWB radios [9], [12], directional antennas [10], [11] etc. Moreover, since the power consumed by the radio frequency (RF) power amplifier of the network interface card (NIC) directly depends on the transmission power, there are a lot of research efforts [13]-[15], for e.g., COMPOW, that focus on designing power-optimal algorithms for maximizing the lifetime of the energy-constrained nodes such as sensor nodes. Surprisingly, there are opposite arguments recently [16]-[18]. In [16]-[18], researchers study the power problem by various approaches such as optimization, simulations etc and show that under some network configurations capacity can be indeed maximized by properly increasing the transmission power.

While the current state of art [2]-[6],[9]-[15] resort to utilizing minimum transmission power for maximizing spatial reuse, lifetime of energy-constrained nodes etc on single channel single-radio network (SC-SR) wireless network, we focus our study on the following aspects: (i) It is not understood yet the effect of employing power on multi-channel multi-radio (MC-MR) wireless networks and hence, it is worth investigating whether we can extract more capacity from MC-MR networks by exploiting power. This inspiration indeed comes from the fact that when the number of available channels c is larger than c_{min} (the minimum number of channels needed to achieve conflict-free transmissions in a network) i.e., $c > c_{min}$, the distance between co-channels can be enlarged by utilizing those extra channels $c - c_{min}$. The larger co-channel distance also implies that each node can increase its transmission power without affecting the spatial reuse.

(ii) As mentioned before, most studies focus on employing minimum power either to maximize spatial reuse [2]-[11] and/or lifetime of energy-constrained nodes [13]-[15] or to minimize the relaying burden on nodes [16]-[18]. However, the crucial parameter that these studies overlook is the signal quality at the receiver i.e., SNR (signal-to-noise ratio). Thus, we plan to exploit power to improve the SNR at the receiver.

According to Shannon-Hartley theorem [1], the capacity (bits/second) is a function of the SNR and hence an increase in SNR may be leveraged to increase the capacity which in turn can lead to a system with greater spectral efficiency. One may also note that a reduced transmission power as well as a larger distance between transmitter-receiver pair can in fact affect the quality of signal at the receiver and correspondingly the capacity. Therefore, in contrast to these existing research efforts we look at the power problem from a different perspective and are summarized as follows: (a) We exploit power to enhance the quality of the signal power at the receiver (i.e., SNR) without sacrificing the spatial reuse or lifetime of nodes; (b) As pointed out in [2], an increase in transmission power implies high interference in the surrounding region and consequently a quadratic reduction in the number of coexisting transmissions. As a result, we investigate the effect of transmission power on multi-channel multi-radio (MC-MR) wireless networks; and (c) Finally and most importantly, an increase in transmission power also implies increased energy consumption and hence a reduction in the lifetime of the nodes. Therefore, in this work we focus our attention on networks that have no power constraints such as Wireless Mesh Networks (WMNs) (see CellNode M100 [19]).

Motivated by these ideas, we specifically address the following two questions: (a) Can we maximize the capacity of MC-MR wireless networks by increasing the transmission power? (b) Under what criteria can we increase the transmission power of the nodes in a MC-MR network? The results derived in this paper indicate that it may be possible to build capacity-optimal wireless networks by exploiting the transmission power in MC-MR network under the following two constraints: (i) $c_{min} = O(c)$ and $c = O(nm)$; (ii) $c = \Omega(nm)$—where c_{min} is the minimum number of channels required to achieve conflict-free transmissions in the network and c is the number of existing channels. Furthermore, surprisingly our results also point out that if the maximum transmit power is $P_0 n^{\frac{\alpha}{2}}$ i.e., $P_{max} = P_0 n^{\frac{\alpha}{2}}$, then partitioning the fixed bandwidth into $\Theta(nm)$ subchannels leads to higher resource utilization and thus enhance the capacity of the wireless network. This indeed is an interesting and contrasting result to previously published works such as in [7].

In summary, the main contributions of this paper are the following:

- This paper produces the first effort to quantify the effect of employing power on the capacity of multi-radio wireless network operating on multiple channels.
- Since the key objective of this paper is to determine the optimal capacity region, we derive the upper and lower bounds on the capacity of the proposed model (MC-MR

networks exploiting power) when node placements are chosen arbitrarily.
- Contrasting to existing literatures, we derive the bounds on capacity for the proposed model to show that unlike single channel wireless networks, MC-MR wireless networks in fact allow us to exploit power to obtain higher capacity and hence it may be possible to build capacity-optimal MC-MR networks by utilizing power.
- Furthermore, we show that the previously established bounds in [7] for multi-channel multi-radio (MC-MR) networks[1] may not be accurate and thus presents necessary modifications to obtain more accurate results.

Definitions:

(a) Arbitrary network: We study the capacity of the proposed model under the arbitrary setting introduced by Gupta and Kumar [2]. In the arbitrary network setting, we suppose that n nodes are arbitrarily located in a torus of unit area in the plane. Each node has an arbitrarily chosen destination to which it sends traffic at an arbitrary rate. Each node can chose an arbitrary range or power level for each transmission. Specifically, since the location of nodes, traffic pattern can be controlled in arbitrary setting, the bounds obtained for this scenario are applicable to any network and may be viewed as the best case bounds on network capacity especially for static multihop networks such as WMNs.

(b) Transport Capacity: We study the transport capacity of the network which is defined as the sum of the distances toward the destination traveled by every bit per unit time. Hence the transport capacity is measured in bit-meters per second. For instance, let $X_{R(k)}$ be the destination for the flow from node X_k. If $b_k(\tau)$ bits reach from node X_k to receiver $X_{R(k)}$ in τ seconds, then $\sum_{k=1}^{n} b_k(\tau) \left\| X_k - X_{R(k)} \right\|$ bit-meters are transported in τ seconds, where $\|x - y\|$ denote the Euclidean distance between x and y. The transport capacity of the network, C_T, is given by $\lim_{\tau \to \infty} \frac{1}{\tau} \sum_{k=1}^{n} b_k(\tau) \left\| X_k - X_{R(k)} \right\|$ bit-meters per second.

(c) Notations: We use the following asymptotic notation to represent the bounds.

- $f(n) = O(g(n))$ implies that $\exists k, N$, such that $f(n) \leq kg(n)$ for $n > N$.
- $f(n) = o(g(n))$ implies that $\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$.
- $f(n) = \omega(g(n))$ implies that $g(n) = o(f(n))$.
- $f(n) = \Omega(g(n))$ implies that $g(n) = O(f(n))$.
- $f(n) = \Theta(g(n))$ implies that $\exists k_1, k_2, N$, such that $k_1 g(n) \leq f(n) \leq k_2 g(n)$ for $n > N$.

The rest of the paper is organized as follows: Section II discusses the motivation behind this work. In section III, we initially start with the discussion of the network model and then focus on the models for communication. Section IV presents some useful lemmas used to obtain the capacity bounds of the proposed model and in section V we present the summary of our contributions. In section VI and VII, we
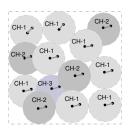
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[1]We refer to MC-MR networks exploiting power as the proposed model and the MC-MR networks using minimum transmission power such as in [7] as the basic model.

establish the capacity of the proposed model under arbitrary settings. Section VIII presents the implications of the results derived in this paper. In section IX, we present the related work. Finally, we summarize our work in section X.

II. Motivation

In this section, we discuss an example to illustrate the power problem in MC-MR networks. For convenience of elucidation, here we consider a single radio multi-channel network as shown in Figure 1 and assume that each transmitter-receiver pair is placed at a distance of $O(1)$ apart. Suppose that c channels are present in the network. Also, let c_{min} be the minimum number of channels needed to achieve the maximum number of simultaneous transmissions in a network ($\frac{n}{2}$) when nodes transmit at the minimum power level say P_0. Let us also assume that $c_{min} < c$.

In the Figure 1(left), we assign each overlapping interference disk with a distinct channel. Since all transmitters in this setting employ power level P_0, the number of channels needed to achieve the maximum simultaneous transmissions is c_{min}. Recall that $c > c_{min}$ channels are present in the network. It should be pretty obvious from the Figure 1(left) that there is no benefit in assigning those remaining $c - c_{min}$ channels to this particular setting since the network has already reached the maximum number of simultaneous transmissions. Thus the question arise, how could we exploit the residual $c - c_{min}$ channels? What if we allow each transmitter to increase its power from P_0 to say $P > P_0$ such that all the channels— that is c— present in the network are completely utilized. We show this scenario in Figure 1(right) where each node increase its transmission power from P_0 to the power P where all the existing c channels are exploited.[2] Next we determine the
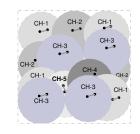

Fig. 1. Figure on the left shows transmitter-receiver pairs transmit at a minimum power level P_0 and utilizing c_{min} channels. While figure on the right shows the same setting when nodes transmit at a power level $P > P_0$ and utilizing c channels. Each circle (or disk) corresponds to the interference disk around a transmitter-receiver pair.

transport capacity, measured in bits-meters/sec, of the multi-channel network transmitting at power levels P_0 and $P > P_0$. Let B_{P_0} and B_P be the bit rate between each transmitter-receiver pair employing power levels P_0 and P respectively. Since each overlapping disk is assigned a distinct channel under both settings, we have $\frac{n}{2}$ simultaneous transmissions and hence the network capacity for transmit power level P_0 and P are $B_{P_0}\frac{n}{2}$ and $B_P\frac{n}{2}$ respectively. Based on Shannon-Hartley

[2]note that an increase in transmission power implies an increase in interference disk.

theorem, we know that an increase in transmission power implies high data rate and hence we have $B_P\frac{n}{2} > B_{P_0}\frac{n}{2}$. As this example indicates, when the available number of channels c is larger than c_{min}, we can still extract more capacity from multi-channel network by exploiting power. This indeed is the motivation behind this work.

III. Background

The objective of this paper is to derive the bounds on the capacity of the proposed model when node placements are chosen arbitrarily. For the ease of exposition, we start with the discussion of the network model and then focus on the models for communication.

A. Network Model and Assumptions

Consider a network of n nodes in a torus of unit area. Let X_i, $1 \leq i \leq n$, denote the location of node i. We will use X_i to denote a node as well as its location. Let $\left\{(X_i, X_{R(i)}) : i \in T_X\right\}$ be the set of all transmitter-receiver pairs in some particular slot and P_i be the transmitted power levels for these pairs. Similar to [2], we also assume a slotted model for convenience of elucidation. Let the transmission radius and interference radius be denoted as $r(i)$ and $Ir(i)$. We further take the following assumptions.

- We assume that there are c channels in the network and each node is equipped with m interfaces, $1 \leq m \leq c$. Moreover, we assume that all nodes transmit on an ideal channel without channel fading.
- We also assume that an interface is capable of transmitting or receiving data on any one channel at a given time.
- We assume that each node $X_i : i \in T_X$ is constrained to a maximum transmit power P_{max} such that $P_0 \leq P_i \leq P_{max}$, where P_0 is the minimum transmit power.

B. Impact of Power on Interference Model

We study the capacity of the proposed model under the so-called model of communication, the protocol model. In this model, the transmission from node $X_i, i \in T_X$, is successfully received by the receiver $X_{R(i)}$ if and only if the receiving node $X_{R(i)}$ is in the transmission radius of the corresponding transmitting node X_i and is out of the interference radius of all other transmitting nodes $X_k, k \in T_X \setminus i$. In [2], Gupta and Kumar do not explicitly take into account the dependency of power of each node on the interference under the protocol model. Thus, we modify the interference model by considering the power level of each node. To derive the necessary and sufficient condition for a successful transmission, we first quantify the transmission and interference radius of a node in the wireless network as in [3].

1) Transmission and Interference Radii: From the theory of communication, we know that both the transmission and interference radius of a node in fact depends on two factors— transmission power and the propagation gain. Thus, for a transmission from node X_i to its receiver $X_{R(i)}$, we employ the following widely used model for propagation gain, $g_{i,R(i)} = (\left\|X_i - X_{R(i)}\right\|)^{-\alpha}$, where $\alpha > 2$ is the path loss

exponent and $\left\| X_i - X_{R(i)} \right\|$ is the physical distance between transmitter-receiver pair X_i and $X_{R(i)}$. In this context, we assume that a data transmission from node X_i to receiver $X_{R(i)}$ is successful only if the received signal strength at $X_{R(i)}$ exceeds a power threshold, say η i.e., $\frac{P_i}{(\left\| X_i - X_{R(i)} \right\|)^\alpha} \geq \eta$. Then, the transmission radius of the node, denoted as $r(i)$ is:

$$r(i) = (\frac{P_i}{\eta})^{1/\alpha}$$

Similarly, we assume that a transmission from node X_i is successfully received at $X_{R(i)}$ only if the interference power level does not exceed a threshold, say β at the receiver. Following the same derivation for the transmission radius, the interference radius of a node, denoted as $Ir(i)$ is obtained as follows:

$$Ir(i) = (\frac{P_i}{\beta})^{1/\alpha}$$

2) Necessary and Sufficient Condition for Successful Transmission: For a given channel $x \in c$, we present the necessary and sufficient conditions to schedule a successful transmission from node X_i to its receiver node $X_{R(i)}$ under the protocol model.

1) The receiving node $X_{R(i)}$ must be physically within the transmission radius of node X_i i.e.,

$$\left\| X_i - X_{R(i)} \right\| \leq r(i) = (\frac{P_i}{\eta})^{1/\alpha} \tag{1}$$

2) The receiving node $X_{R(i)}$ should lie outside the interference radius of any other node $k \in T_X \setminus i$ that is transmitting in the same channel, i.e.,

$$\left\| X_k - X_{R(i)} \right\| \geq Ir(k) = (\frac{P_k}{\beta})^{1/\alpha} \tag{2}$$

C. Data Rate Model

We use Shannon's capacity formula for the additive white Gaussian noise channel to model the data rate. In this model, the data rate is a function of the signal-to-interference-plus-noise ratio (SINR) at the receiver. In this case the data rate from transmitter X_i to its receiver $X_{R(i)}$ is given in bits/sec by

$$B_i = W_x \log_2(1 + \frac{\frac{P_i}{(\left\| X_i - X_{R(i)} \right\|)^\alpha}}{N_0 W_x + \sum_{k \in T_X, k \neq i} \frac{P_k}{(\left\| X_k - X_{R(i)} \right\|)^\alpha}}) \tag{3}$$

where W_x is the bandwidth of the channel $x \in c$ in hertz, and $\frac{N_0}{2}$ is the noise spectral density in watts/hertz. However, recall that we employ protocol model to characterize the interference and hence under the so-called protocol model for each receiver $X_{R(i)}$ if all the transmitters $X_k \in T_X \setminus i$ are more than $Ir(k)$ distance away, then the accumulated interference at $X_{R(i)}$ will be negligible. Hence, equation 3 can be rewritten as

$$B_i = W_x \log_2(1 + \frac{\frac{P_i}{(\left\| X_i - X_{R(i)} \right\|)^\alpha}}{N_0 W_x}) \tag{4}$$

Moreover, recall that in subsection III-A, we assume that each node is constrained to a maximum transmit power

P_{max} such that $P_0 \leq P_i \leq P_{max}$, where P_0 is the minimum transmit power. For the simplicity of analysis, let minimum (P_0) and maximum (P_{max}) transmit power level be $N_0 W_x \eta \left\| X_i - X_{R(i)} \right\|^\alpha$ and $P_0 n^{\frac{\alpha}{2}} = N_0 W_x \eta (\sqrt{n} \left\| X_i - X_{R(i)} \right\|)^\alpha$ respectively in the sequel.

D. Assumptions on Channel Model

As in [7], we consider the following basic assumptions on the channel model.

- Channel Model A: In this model, we have $W_x = \frac{W}{c}$ for all $x \in c$. Intuitively, we can see that as the number of channels (c) increases, the bandwidth for each channel decreases and hence the data rate supported by each channel will be less (see eq (4)). This model is applicable to scenarios where the total available bandwidth is fixed and new channels are created by splitting the existing channels.

- Channel Model B: In this model, we have $W_x = W$ for all $x \in c$. Intuitively, we can observe that each channel has a bandwidth of W and hence, this model is applicable to scenarios where new channels are created by utilizing additional frequency spectrum.

The derivation of the proof is based on the assumption of channel model A. However, all the results under channel model B can be obtained by replacing W by Wc.

IV. SOME USEFUL RESULTS

In this section, we derive some results that is used to obtain the upper bound of the proposed model.

Lemma 1. Receiver Interference Model: In a wireless network under protocol interference model, let $(X_i, X_{R(i)})$ and $(X_k, X_{R(k)})$ be two simultaneous active transmitter-receiver pairs over the same channel, then a disk of radius $\frac{\Delta}{2}(\left\| X_i - X_{R(i)} \right\|)$ centered at $X_{R(i)}$ and a disk of radius $\frac{\Delta}{2}(\left\| X_k - X_{R(k)} \right\|)$ centered at $X_{R(k)}$, where $\Delta = (\frac{k_3 \eta}{\beta})^{\frac{1}{\alpha}} - 1$, must be disjoint.

Proof: Let $(X_i, X_{R(i)})$ and $(X_k, X_{R(k)})$ be two active transmitter-receiver pairs. From equation 1, we can compute P_i and P_k as $\geq \eta \left\| X_i - X_{R(i)} \right\|^\alpha$ and $\geq \eta \left\| X_k - X_{R(k)} \right\|^\alpha$ respectively. Thus, the preceding equations can be rewritten as follows:

$$P_i = k_3 \eta \left\| X_i - X_{R(i)} \right\|^\alpha \text{ and } P_k = k_3 \eta \left\| X_k - X_{R(k)} \right\|^\alpha \tag{5}$$

where $k_3 \geq 1$ is a constant. Recall that the protocol model places the following constraints on the relative locations of these nodes and using equation 2, we get:

$$\left\| X_k - X_{R(i)} \right\| \geq (\frac{k_3 \eta}{\beta})^{\frac{1}{\alpha}} \left\| X_k - X_{R(k)} \right\| \tag{6}$$

$$\left\| X_i - X_{R(k)} \right\| \geq (\frac{k_3 \eta}{\beta})^{\frac{1}{\alpha}} \left\| X_i - X_{R(i)} \right\| \tag{7}$$

Using the triangle inequality first, we can derive the following relation between the location of the node pairs $(X_i, X_{R(i)})$

and $(X_k, X_{R(k)})$.

$$\left\| X_{R(i)} - X_{R(k)} \right\| + \left\| X_k - X_{R(k)} \right\| \geq \left\| X_{R(i)} - X_k \right\|$$

$$\left\| X_{R(i)} - X_{R(k)} \right\| \geq \left(\frac{k_3 \eta}{\beta} \right)^{\frac{1}{\alpha}} \left\| X_k - X_{R(k)} \right\| - \left\| X_k - X_{R(k)} \right\|$$
(8)

Similarly we can write,

$$\left\| X_{R(i)} - X_{R(k)} \right\| \geq \left(\frac{k_3 \eta}{\beta} \right)^{\frac{1}{\alpha}} \left\| X_i - X_{R(i)} \right\| - \left\| X_i - X_{R(i)} \right\|$$
(9)

Adding the inequalities in (8) and (9), we obtain

$$\left\| X_{R(i)} - X_{R(k)} \right\| \geq \frac{\Delta}{2} \left(\left\| X_i - X_{R(i)} \right\| + \left\| X_k - X_{R(k)} \right\| \right)$$

where $\Delta = \left(\frac{k_3 \eta}{\beta} \right)^{\frac{1}{\alpha}} - 1$. Note that $X_{R(i)}$ and $X_{R(k)}$ are the receivers and we can deduce this inequality to say that for transmitter-receiver pairs $(X_i, X_{R(i)})$ and $(X_k, X_{R(k)})$ to be active, then a disk of radius $\frac{\left(\frac{k_3 \eta}{\beta} \right)^{\frac{1}{\alpha}} - 1}{2} (\left\| X_i - X_{R(i)} \right\|)$ centered at $X_{R(i)}$ and a disk of radius $\frac{\left(\frac{k_3 \eta}{\beta} \right)^{\frac{1}{\alpha}} - 1}{2} (\left\| X_k - X_{R(k)} \right\|)$ centered at $X_{R(k)}$ should not overlap. We term this model as the receiver interference model. ∎

It turns out from Lemma 1 that the radius of the interference disk centered around each receiver directly depends on Δ, where $\Delta = \left(\frac{k_3 \eta}{\beta} \right)^{\frac{1}{\alpha}} - 1$. Recall from section III-C that power levels P_0 and P_{max} are set as $N_0 W_m \eta \left\| X_i - X_{R(i)} \right\|^\alpha$ and $N_0 W_m n^{\frac{\alpha}{2}} \eta \left\| X_i - X_{R(i)} \right\|^\alpha$ respectively. From equation 5, it also follows that $P_i = k_3 \eta \left\| X_i - X_{R(i)} \right\|^\alpha$, $\forall i \in T_X$. This implies that $k_3 \geq 1$ takes values $k_2 N_0 W_m$ and $k_2 N_0 W_m n^{\frac{\alpha}{2}}$ respectively for transmit power levels P_0 and P_{max}, where k_2 is a constant. Since the radius of the interference disk directly depends on Δ, we note that an increase in transmit power can in fact lead to an increase in the radius of the disk.

Lemma 2. For $z \in R^+$ and $a \geq 1$, $\ln(1 + z^a) \leq az$

Proof:

$$\begin{aligned} \ln(1 + z^a) &\leq \ln((1 + z)^a) \\ &= a \ln(1 + z) \leq az \end{aligned}$$

∎

V. MAIN RESULT

In this section, we present the capacity bounds of MC-MR networks exploiting power for the following scenarios: (a) $c_{min} < c < nm/2$; (b) $c_{min} \geq c$ and (c) $c \geq nm/2$. Under the receiver-based interference model and when node placements are chosen arbitrarily, the transport capacity of the proposed model measured in bit-meters/second under channel models A and B is given by:

- Channel Model A:

$$:= \begin{cases} \Theta(W \sqrt{\frac{nm}{c_{min}}}) & \text{when } c_{min} < c < nm/2; \\ \Theta(W \sqrt{\frac{nm}{c}}) & \text{when } c_{min} \geq c; \\ \Theta(\frac{Wn^2m}{c}) & c \geq nm/2; \end{cases}$$

- Channel Model B:

$$:= \begin{cases} \Theta(Wc \sqrt{\frac{nm}{c_{min}}}) & \text{when } c_{min} < c < nm/2; \\ \Theta(W \sqrt{nmc}) & \text{when } c_{min} \geq c; \\ \Theta(Wn^2m) & c \geq nm/2; \end{cases}$$

VI. AN UPPER BOUND ON TRANSPORT CAPACITY

We consider n nodes arbitrarily located in a torus of unit area on the plane. The following are the basic assumptions on the model being considered:

1) The network transports λnT bits over T seconds.
2) The average distance between the source and destination of a bit is $\overline{L}$. Together with (1), this implies that a bit-meters/second of $\lambda n \overline{L}$ is achieved.
3) Transmissions are slotted into synchronized slots of length τ seconds.
4) There are c channels present in the network and each node has m ($1 \leq m \leq c$) half-duplex interfaces (or radios). Moreover, we assume that each node in the network employ the same transmit power in all channels. This model is referred to as *symmetric allocation*. In Appendix, we also analyze a different setting where all nodes do not employ the same transmit power which is referred to as *asymmetric allocation*.
5) There also exists minimum number of channels $c_{min} \leq c$ or $c_{min} \geq c$ with which the network can achieve interference-free transmissions.
6) When the domain is of A square meters rather than 1 square meters, then all the upper bounds are scaled by $\sqrt{A}$.

Theorem 1. Consider a wireless network with n nodes each with m half-duplex interfaces deployed in a domain of unit area under the receiver-based interference model, then the transport capacity $\lambda n \overline{L}$ is upper bounded as follows:
Channel Model A:

- when $c_{min} < c < nm/2$

$$\lambda nT\overline{L} \leq \frac{\sqrt{2}k_5 WT}{\Delta} \sqrt{\frac{nm}{\pi c_{min}}} \text{ bit-meters/second}$$

- when $c_{min} \geq c$

$$\lambda nT\overline{L} \leq \frac{\sqrt{2}k_6 WT}{\Delta} \sqrt{\frac{nm}{\pi c}} \text{ bit-meters/second}$$

where $k_5 = (\eta^{\frac{2}{\alpha}} \frac{\alpha}{2})/(\ln 2)$ and $k_6 = \eta/(\ln 2)$.

Proof: Consider an arbitrary bit b, where $1 \leq b \leq \lambda nT$. Let the number of hops that bit b traverses from its origin to its destination in a sequence of $h(b)$ hops, where the hth hop traverses a distance of d_b^h. Then from assumption (2), we have

$$\sum_{b=1}^{\lambda nT} \sum_{h=1}^{h(b)} d_b^h \geq \lambda nT\overline{L}$$
(10)

which follows from the fact that the line segment has the shortest length among all curves passing two given points.

Let H be the total number of hops traversed by all bits in T seconds i.e., $H = \sum_{b=1}^{\lambda nT} h(b)$. Since each node has m

interfaces, and each interface transmits over a channel with rate B bits per second [3], the total number of bits that can be transmitted by all nodes over all interfaces is at most $\frac{BTnm}{2}$ (where the factor $1/2$ is based on the half-duplex nature of the interface). Hence, we have

$$H = \sum_{b=1}^{\lambda nT} h(b) \le \frac{BTnm}{2} \tag{11}$$

From Lemma (1), we know that the disks of radius $\frac{\Delta}{2}$, where $\Delta = (\frac{k_3\eta}{\beta})^{\frac{1}{\alpha}} - 1$, times the length of hops centered at the receivers over the same channel are essentially disjoint. This may be viewed as each hop consuming a disk of radius $\frac{\Delta}{2}$ times the length of the hop around each receiver. Since the area consumed on each channel is bounded above by the area of the domain ($= 1m^2$), summing over all channels and the slots (there can be no more than $\frac{T}{\tau}$ slots in T seconds) we have the following constraint at equation (12):

Remark 1. We are particularly interested in the scenario where $c_{min} < c < mn/2$. In [7], the authors assume that c channels are present in the network and hence the summation of area consumed on each channel, $x \in c$, sum to 1. However, this reasoning is not accurate. Recall that there also exists a minimum number of channels, c_{min} where $c_{min} < c$ or $c_{min} > c$, with which the network can achieve conflict-free transmissions. Hence $c_{min} < c$ implies that the area consumed on each channel is upper bounded by $O(\frac{c_{min}}{c})$. This also implies that if each node increases its disk radius by $\sqrt{\frac{c}{c_{min}}}$, then all existing c channels can be exploited. On the other hand, note that the reasoning of [7] holds true when $c_{min} \ge c$.

$$\sum_{b=1}^{\lambda nT} \sum_{h=1}^{h(b)} \pi(\frac{\Delta}{2})^2 (d_b^h)^2 \le BT\min(c_{min}, c) \tag{12}$$

which can be rewritten as

$$\sum_{b=1}^{\lambda nT} \sum_{h=1}^{h(b)} \frac{1}{H}(d_b^h)^2 \le \frac{4BT\min(c_{min}, c)}{\pi(\Delta)^2} \tag{13}$$

Noting that the quadratic equation is convex, we have

$$(\sum_{b=1}^{\lambda nT} \sum_{h=1}^{h(b)} \frac{1}{H}d_b^h)^2 \le \sum_{b=1}^{\lambda nT} \sum_{h=1}^{h(b)} \frac{1}{H}(d_b^h)^2 \tag{14}$$

Combining (13) and (14) yields

$$\sum_{b=1}^{\lambda nT} \sum_{h=1}^{h(b)} d_b^h \le \sqrt{\frac{4BTH\min(c_{min}, c)}{\pi(\Delta)^2}} \tag{15}$$

Now substituting (10) and (11) in (15) yields the result

$$\lambda nT\overline{L} \le \frac{\sqrt{2}BT}{\Delta}\sqrt{\frac{nm\min(c_{min}, c)}{\pi}} \tag{16}$$

[3]Since nodes employ same power (see assumption (4)), we assume that each node transmit over the channel with same data rate B i.e., $B_i = B \forall i \in T_X$

To get the final capacity bounds, we need to consider the following scenarios where different values for data rate B are obtained.

CASE A: $c_{min} < c < mn/2$. Recall that while c channels are present in the network, no more than c_{min} channels are utilized. Hence to completely exploit the existing c channels, each node can increase the disk size by $\sqrt{\frac{c}{c_{min}}}$ which in turn implies that each node can augment the power P from P_0 to $P_0(\sqrt{\frac{c}{c_{min}}})^\alpha$. Substituting $P_0(\sqrt{\frac{c}{c_{min}}})^\alpha$ in equation (4) and using Lemma 2, we get[4]

$$B = W_x \log_2(1 + \eta(\sqrt{\frac{c}{c_{min}}})^\alpha) = W_x \log_2(1 + (\eta^{\frac{2}{\alpha}}\frac{c}{c_{min}})^{\frac{\alpha}{2}})$$
$$= \frac{W_x}{\ln(2)}\ln(1 + (\eta^{\frac{2}{\alpha}}\frac{c}{c_{min}})^{\frac{\alpha}{2}})$$
$$\le \frac{W_x}{\ln(2)}\eta^{\frac{2}{\alpha}}\frac{\alpha}{2}(\frac{c}{c_{min}}) = W_x k_5 \frac{c}{c_{min}} \tag{17}$$

where $k_5 = (\eta^{\frac{2}{\alpha}}\frac{\alpha}{2})/(\ln 2)$. Since $\frac{\alpha}{2} \ge 1$, we can use Lemma 2 to replace $\ln(1+x)^{\frac{\alpha}{2}}$ by $(\alpha/2)x$.

Substituting $B = W_x k_5 \frac{c}{c_{min}}$ in (16) and $W_x = W/c$, we get

$$\lambda nT\overline{L} \le \frac{\sqrt{2}k_5 WT}{\Delta}\sqrt{\frac{nm}{\pi c_{min}}} \tag{18}$$

CASE B: $c_{min} \ge c$. In this setting when minimum number of channels, c_{min}, is greater than the existing channels i.e., $\min(c_{min}, c) = c$, each node can transmit only at the minimum power level P_0. Hence substituting P_0 in equation (4) and using Lemma 2, we get $B = \frac{W_x}{\ln 2}\eta$. Now, combining $B = \frac{W_x}{\ln 2}\eta$ and $W_x = W/c$ in 16, we obtain

$$\lambda nT\overline{L} \le \frac{\sqrt{2}k_6 WT}{\Delta}\sqrt{\frac{nm}{\pi c}} \tag{19}$$

where $k_6 = \eta/(\ln 2)$. ∎

Remark 2. *We can see that in the scenario where $c_{min} < c < mn/2$, the capacity of the arbitrary network under minimum power level P_0 (i.e., basic model), is*

$$\lambda nT\overline{L} \le \frac{\sqrt{2}k_6 WT}{\Delta c}\sqrt{\frac{nmc_{min}}{\pi}} \tag{20}$$

Clearly, we can observe that ignoring the constants the proposed model has a gain of $\frac{c}{c_{min}}$ over the basic model under the constraint $c_{min} < c < mn/2$.

Theorem 2. Consider a wireless network with n nodes each with m half-duplex interfaces deployed in a domain of unit area and when $c \ge mn/2$, then the transport capacity is upper bounded as follows:

- **Channel Model A**

$$\lambda nT\overline{L} \le \frac{Wk_5 n^2 m}{2c} \text{ bit-meters/second}$$

[4]we used the following $\log_2(x) = \frac{\log_e(x)}{\log_e(2)}$ to replace $\log_2(x)$ by $\log_e(x)$.

- **Channel Model B**

$$\lambda n T \overline{L} \leq \frac{W k_5 n^2 m}{2} \text{ bit-meters/second}$$

where $k_5 = (\eta^{\frac{2}{\alpha}} \frac{\alpha}{2})/(\ln 2)$.

Proof: The capacity of arbitrary networks is also constrained by the maximum number of bits that can be transmitted simultaneously over all interfaces in the network. Since each node has m interfaces, there are total of nm interfaces in the network. Each interface can transmit at a data rate of B bits per second. Also, the maximum distance a bit can travel in the network is $O(1)$ meters. Hence, the total network capacity is at most $O(B \frac{nm}{2})$ bit-meters/sec.

However, noting that the maximum number of simultaneous transmissions feasible in this network is $nm/2$ and hence when $c \geq nm/2$ channels are present, indeed each node can tune its interface to a different channel. This as well as implies that each node can transmit to a maximum power level, $P_{max} = P_0(\sqrt{n})^\alpha$, without causing interference to other transmissions in the network. Therefore using P_{max} in equation (4) and employing Lemma 2, we get $B = W_x k_5 n$. Now, combining $B = W_x k_5 n$ and $W_x = W/c$ in $O(B \frac{nm}{2})$, the network capacity is obtained as $O(\frac{W k_5 n^2 m}{2c})$. ∎

Remark 3. *We can see that in the scenario where $c \geq mn/2$, the capacity of the arbitrary network under minimum power level P_0 (i.e., basic model), is $O(\frac{W k_6 nm}{2c})$. Indeed, we can observe that ignoring the constants the proposed model has a gain of n over the basic model under the constraint $c \geq mn/2$.*

VII. A LOWER BOUND ON TRANSPORT CAPACITY

We will now manifest that the upper bound obtained in the previous section is indeed sharp under the receiver-based interference model by exhibiting a scenario where it is achieved. Also, note that in this section we provide the lower bound construction for a single interface multi-channel network; however the results can be easily extended to a multi-interface network by using the Lemma from [7].

Theorem 3. When $c < n/2$, there is a placement of nodes and an assignment of traffic patterns such that the network can achieve $\frac{W}{\Delta\sqrt{2}} \sqrt{\frac{n}{c_{min}}}$ bit-meters/sec for $\min(c_{min}, c) = c_{min}$ and $\frac{W}{\Delta\sqrt{2}} \sqrt{\frac{n}{c}}$ bit-meters/sec for $\min(c_{min}, c) = c$ respectively under channel model A.

Proof: We consider a torus of unit area. Partition the area into $\frac{n}{2c}$ equal-sized square cells, and place c transmitter-receiver pairs in each cell. Intuitively, this implies that transmitter-receiver pairs operating in distinct channels can be placed at the same location. The location of the transmitter in each cell is shown in Figure 2. Since the area of the domain is $1m^2$, each cell has an area of $\frac{2c}{n}$ and sides of length $s = \sqrt{\frac{2c}{n}}$. Let $k = \frac{c}{\min(c_{min}, c)}$. Each transmitter in a cell is placed at a distance of $(2 + \sqrt{k}\Delta)d$ from the transmitter at its adjacent cell (recall the receiver-based interference model given by Lemma 1). For instance, we can see that
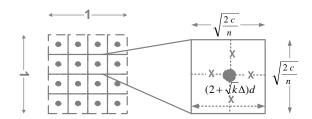

Fig. 2. Figure on the left shows the arrangement of the transmitters in the domain of area = $1m^2$. The distance between every transmitter is $(2+\sqrt{k}\Delta)d$ and the receiver of the corresponding transmitter can be placed at any of the locations marked X which is at a distance of d from the transmitter. From this arrangement, the distance between two receivers is $(\sqrt{k}\Delta)d$ and hence, according to Lemma 1, the disks of radius $(\sqrt{k}\Delta)\frac{d}{2}$ centered around each receiver do not overlap.

when $\min(c_{min}, c) = c$, the distance between two adjacent transmitters is $(2 + \Delta)d$ which corresponds to the diameter of the interference disk when nodes transmits at a power level P_0.

From the above construction, it can be verified that there are total of $\frac{n}{2c} \times c = \frac{n}{2}$ simultaneous transmitter-receiver pairs located within the domain, each transmitting at a rate of $\frac{W}{c}(k)$ over a distance of $d = \frac{s}{(2+\sqrt{k}\Delta)}$. Hence the total capacity of the network is $\frac{W(k)}{c} \frac{n}{2} d = \frac{W(k)}{c} \frac{n}{2} \frac{s}{(2+\sqrt{k}\Delta)}$ bit-meters/sec. Recall that $k = \frac{c}{\min(c_{min}, c)}$. Therefore substituting for k, we obtain the network capacity as $\approx \frac{W}{\Delta\sqrt{2}} \sqrt{\frac{n}{c_{min}}}$ for $\min(c_{min}, c) = c_{min}$ and $\approx \frac{W}{\Delta\sqrt{2}} \sqrt{\frac{n}{c}}$ for $\min(c_{min}, c) = c$. ∎

Theorem 4. When $c \geq n/2$, there is a placement of nodes and an assignment of traffic patterns such that the network can achieve $\frac{Wn^2}{2c}$ bit-meters/sec under channel model A.

Proof: Recall that the domain is a torus of unit area. Let $g = \min(c, \frac{n}{2})$. Partition the area into $\frac{n}{2g}$ cells, and place g transmitter-receiver pairs in the cell. Intuitively, we can see that since $c \geq n/2$, each g transmitter-receiver pair will be operating on a distinct channel and hence each pair can be placed at the same location. Since the area of the domain is $1m^2$, each cell has an area of $\frac{2g}{n}$ and sides of length $s = \sqrt{\frac{2g}{n}}$. Recall that $g = \min(c, \frac{n}{2})$ and $c \geq n/2$, hence length of the square cell is $s = 1$. Each transmitter is placed at the center of the edge of the cell and its corresponding receiver is placed at the center of the opposite edge at a distance of d.

From the above construction, it can be verified that there are total of $\frac{n}{2g} \times g = \frac{n}{2}$ simultaneous transmitter-receiver pairs located within the domain, each transmitting at a rate of $\frac{W}{c}(P_{max}) \approx \frac{Wn}{c}$ over a distance of $d = s = 1$. Hence the total capacity of the network is $\frac{Wn}{c} \frac{n}{2} d = \frac{Wn^2}{2c}$ bit-meters/sec ∎

VIII. DISCUSSIONS

In Table I we study the gain of the MC-MR network exploiting power over the basic model under the constraint $c_{min} = O(c)$ and $c = O(n)$. Indeed, we can see that when

the number of channels existing in the network, c, exceeds the minimum channels required for achieving conflict-free transmissions i.e., c_{min}, the proposed model has a gain of $\frac{c}{c_{min}}$ over the basic model. However, when $c_{min} \geq c$ we see that there is no point in increasing the transmit power level and consequently end with the same capacity as in [7]—i.e., gain = $\Theta(1)$. Therefore, one implication that follows from the results of this paper is to achieve a capacity gain of $\frac{c}{c_{min}}$, the designers should perhaps want to consider the constraint $c_{min} < c$ while developing energy-efficient networks such as mesh networks.

TABLE I
GAIN OF THE PROPOSED MODEL OVER BASIC MODEL ($\frac{c}{c_{min}}$) FOR CASE $m = 1, c = O(n)$

$c = O(n)$	$c_{min} = 1$	$c_{min} = \log \log n$	$c_{min} = \log n$
1	$\Theta(1)$	$\Theta(1)$	$\Theta(1)$
$\log \log n$	$\Theta(\log \log n)$	$\Theta(1)$	$\Theta(1)$
$\log n$	$\Theta(\log n)$	$\Theta(\frac{\log n}{\log \log n})$	$\Theta(1)$

Yet another issue that arises is whether $\frac{c}{c_{min}}$ is indeed the maximum gain achievable under the constraint $c_{min} \leq c$. For instance, consider the network of three transmitter-receiver pairs in Figure 3. Suppose that $c_{min} = c = 2$, then according to our approach each pair is constrained to a minimum transmit power level, P_0 and hence the network capacity in bits/second is $W_x(P_0)\frac{n}{2}$. However, we can see that even though $c_{min} = c$, transmitter-receiver pair operating on channel 2 can indeed transmit at maximum power level P_{max} without causing interference to other transmissions; Thus, in this setting the network can achieve a capacity of $W_x(P_0)\frac{n-2}{2} + W_x(P_{max})$ bits/second which is obviously greater than the capacity at constant transmit power setting. Certainly, there is still room for capacity improvement and thus, we plan to focus on this topic for our future research work. Further, this may also offer some suggestive guidelines for designers of static mesh network. Correspondingly in Table II, we study the capacity of
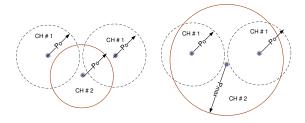

Fig. 3. Figure on the left shows that all transmitters transmit at a constant minimum power level P_0 while figure on the right shows the setting where some nodes can transmit at power level $> P_0$. Each circle corresponds to the interference disk around a transmitter-receiver pair.

both proposed and basic models for the case $m = 1, c = \Omega(n)$. One interesting situation that arises from the result in Table II is that when $c = n$, even though the channels are partitioned into c subchannels, the transport capacity of the network under the proposed model is $\Theta(Wn)$ i.e, the end-to-end throughput available for each node is $\Theta(W)$. This is a contrasting result

to [7] which claims that the end-to-end throughput of each node under channel partition approaches zero when $c \geq n$. This improvement can be elucidated regarding the data rate-SNR (signal-to-noise ratio) relationship in Shannon-Hartley theorem. According to the theorem, the transmission rate i.e., the data rate per unit bandwidth is a function of SNR and hence an increase in SNR may be leveraged to increase the transmission rate. Therefore, while we employ the maximum power level $P_{max} = P_0 n^{\alpha/2}$, the SNR in fact increases with the power i.e., SNR $\propto P_{max}$ and thus we observe increased data rate for each partitioned channel. Alternatively, we can say that the bandwidth is efficiently utilized under the maximum transmit power level which in turn leads to a higher capacity at $c = n$. On the other hand, when $c > n$, the capacity of the arbitrary network indeed decreases with the channel partitioning. Therefore, one implication that follows from this result is that if the maximum transmit power is $P_0 n^{\alpha/2}$ i.e., $P_{max} = P_0 n^{\alpha/2}$, then dividing the fixed bandwidth into $\Theta(n)$ subchannels does not degrade the capacity of the wireless network. This in fact is an interesting and novel result that the designers may perhaps want to consider while designing energy-efficient wireless networks.

TABLE II
CAPACITY OF BASIC MODEL AND PROPOSED MODEL FOR CASE $m = 1, c = \Omega(n)$ AND UNDER CHANNEL MODEL A

$c = \Omega(n)$	Basic Model	Proposed Model
n	$\Theta(W)$	$\Theta(Wn)$
$n \log n$	$\Theta(\frac{W}{\log n})$	$\Theta(\frac{Wn}{\log n})$
n^2	$\Theta(\frac{W}{n})$	$\Theta(W)$

IX. RELATED WORK

In this section, we initially focus our review on literatures that aim to improve the capacity of wireless networks in several ways and then present a few studies concentrated on implementing power control algorithms for wireless networks.

In their landmark work [2], Gupta and Kumar first derived the capacity of ad hoc wireless networks in the limit as the number of nodes grows to an arbitrarily large level. Under this model, the authors indicate that regardless of the initial location of the nodes and traffic pattern in a fixed area, the throughput in bit-meters per second cannot grow faster than $\sqrt{n}$, and for a special arrangement of nodes and traffic pattern a throughput of $\sqrt{n}$ can be achieved. Therefore, the end-to-end throughput available for each node is $O(1/\sqrt{n})$. Gupta and Kumar also demonstrated the existence of a global scheduling scheme achieving $\Omega(1/\sqrt{n \log n})$ for a network with random traffic pattern and random node distribution. This pessimistic result that the end-to-end throughput available to each node approaches zero as the number of nodes n increases motivated many researchers to improve the capacity bounds of wireless networks in several ways by employing mobility, directional antennas, UWB radios etc.

In [4], [5], [6], the results show that mobility can increase the capacity of wireless network. Under the assumption that

nodes are mobile and the position of each node is ergodic with stationary uniform distribution on an open disk, Grossglauser and Tse [5] manifest that when nodes are mobile it is possible to have a constant or $O(1)$ throughput scaling per source-destination pair *w.h.p* at the cost of increased end-to-end delay as the number of nodes in each unit area goes to infinity. Consequently, several researchers analyzed the trade off between delay and capacity in mobile networks[5], [6]. In [7], the authors study the network capacity in the context where the number of interfaces m at each node may be smaller than the number of available channels c. The result shows that the capacity results are a function of channel-to-interface $\frac{c}{m}$ ratio and under arbitrary network setting there are two distinct capacity regions: when $\frac{c}{m}$ is $O(n)$, the network capacity is $\Theta(W\sqrt{nm/c})$ and when $\frac{c}{m}$ is $\Omega(n)$, the network capacity is $\Theta(Wnm/c)$.

In [8], Liu *et. al* employ the infrastructure support to improve the capacity bound. The work considers the scenario where m base stations are placed in a regular hexagonal pattern within the ad hoc network with n nodes and employs two routing strategies: deterministic and probabilistic. Under the deterministic routing strategy, the result shows that if m grows asymptotically slower than $\sqrt{n}$, the maximum throughput capacity is $\Theta(\sqrt{n/\log(\frac{n}{m^2})}W)$ and if m grows faster than $\sqrt{n}$, the maximum capacity is $\Theta(mW)$, which increases linearly with the number of base stations. In the probabilistic routing strategy, if m grows slower than $\sqrt{n/\log n}$, the maximum throughput capacity has the same asymptotic behavior as pure ad hoc network. If m grows faster than $\sqrt{n/\log n}$, the maximum throughput capacity scales as $\Theta(mW)$, which increases linearly with the number of base stations.

Another relevant body of work is employing directional antennas to improve the capacity bounds under the context of single channel and multiple channel networks. Yi *et. al* [10] show that in a random wireless network, the use of directional antennas can improve the network capacity by a factor of $2\pi/\alpha$, where α is the beamwidth for transmitters and by a factor of $2\pi/\beta$, where β is the beamwidth for receivers. Moreover, the results also indicate that if both transmitters and receivers employ directional antenna, the capacity can be improved by a factor of $4\pi^2/\alpha.\beta$. In [11], Dai *et. al* extended the work on [7] on multi-channel multi-radio networks with the consideration of directional antennas.

Recent literatures indicate that employing capacity leverage unlimited bandwidth resources (UWB radios) can also improve the capacity bounds. Negi and Rajeswaran [9] show that when each node is constrained to a maximum transmit power P_0 and capable of utilizing W Hz of bandwidth, the uniform throughput per node is upper bounded by $O((n \log n)^{(\alpha-1)/2})$ and lower bounded by $\Omega(\frac{n^{(\alpha-1)/2}}{(\log n)^{(\alpha+1)/2}})$ under the limiting case when $B \to \infty$. In [12], the authors study the same problem in [9] and tighten both the lower and upper bounds of the network capacity to $\Theta(n^{(\alpha-1)/2})$ and close the gap between the upper and lower bounds that exist in [9].

Recently, power control in wireless networks has been the focus of extensive research. The main objectives of power control schemes is to reduce the total energy consumed in packet delivery and/or increase network throughput by increasing the channel's spatial reuse through the use of low transmission power in the network. In [13], the authors indicate that network capacity decreases significantly with higher transmission power and hence they suggest using the lowest transmission power to maximize the capacity of network. There are a lot of efforts following this suggestion [14],[15], and focus on using lower transmission power to maximize the network capacity. On the other hand, there is an opposite argument recently [16],[17]. In [17], Behzad *et. al* formulated the problem of power control as an optimization problem and showed that network capacity can be maximized by properly increasing the transmission power. Park *et. al* in [16] also showed through simulations that network capacity can occasionally increases with higher transmission power in certain scenarios. In [18], the authors analyzed why network capacity increases or decreases with higher transmission power in different scenarios, by using carrier sensing and minimum hop count in practice and showed that network capacity can be improved with higher transmission power in the networks with a small diameter.

X. Conclusion

In this paper, we have derived the lower and upper bounds on the capacity of MC-MR networks exploiting power in an arbitrary setting. Specifically, we answer the following questions: (a) Can we maximize the capacity of MC-MR wireless networks by increasing the transmission power?; (b) Under what criteria can we increase the transmission power of the nodes in a MC-MR network? The results derived in this paper indicate that it may be possible to build capacity-optimal wireless networks by exploiting the transmission power in MC-MR network under the following two constraints: (i) $c_{min} = O(c)$ and $c = O(nm)$; (ii) $c = \Omega(nm)$—where c_{min} is the minimum number of channels required to achieve conflict-free transmissions in the network and c is the number of existing channels. Furthermore, surprisingly our results also point out that if the maximum transmit power is $P_0 n^{\frac{\alpha}{2}}$ i.e., $P_{max} = P_0 n^{\frac{\alpha}{2}}$, then partitioning the fixed bandwidth into $\Theta(nm)$ subchannels does not degrade the capacity of the wireless network which is an interesting and contrasting result to previously published works such as in [7]. In future, we also plan to study the capacity bounds of the proposed model under random placement of nodes.

Appendix

While deriving the upper bounds for the MC-MR networks exploiting power in section VI, we assume the *symmetric allocation model* and hence the same transmit power level, P $(P_0 \le P \le P_{max})$, for all the nodes; Here, we adopt a different setting termed as *asymmetric allocation* and analyze the upper bounds for the capacity when $c_{min} < c < mn/2$ [5].

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[5] the bounds remain the same for $c \ge mn/2$ and $c_{min} > c$ under both settings

Theorem 5. Given γ ($0 \leq \gamma \leq 1$), the transport capacity of the MC-MR network under *asymmetric allocation* in region $c_{min} < c < mn/2$ when node placements are chosen arbitrarily and the spatial reuse is described by the receiver-based interference model is given by:

$$\frac{\sqrt{2}T}{\Delta} \left(k_6 \gamma \frac{W}{c} + k_5(1-\gamma)\frac{W}{c}\left(\frac{c-\gamma c_{min}}{(1-\gamma)c_{min}}\right)\right) \sqrt{\frac{nmc_{min}}{\pi}}$$

Proof:

Let us suppose that c channels are present in the network and no more than c_{min} channels are exploited. For a given parameter γ ($0 \leq \gamma \leq 1$), we assume that interfaces tuned to γc_{min} channels transmit at power level P_0 and the remaining interfaces tuned to $(1-\gamma)c_{min}$ can be tuned to a different power setting other than P_0. Then, we can interpret the following that since c channels are present in the network, $c - \gamma c_{min}$ channels can be divided among the interfaces tuned to $(1-\gamma)c_{min}$ channels. Therefore to completely exploit the existing $c - \gamma c_{min}$ channels, each node that is tuned to $(1-\gamma)c_{min}$ channels can increase the disk size by $\sqrt{\frac{c-\gamma c_{min}}{(1-\gamma)c_{min}}}$. This in turn implies that each node can augment the power P from P_0 to $P_0(\sqrt{\frac{c-\gamma c_{min}}{(1-\gamma)c_{min}}})^\alpha$. Substituting $P_0(\sqrt{\frac{c-\gamma c_{min}}{(1-\gamma)c_{min}}})^\alpha$ in equation (4) and using Lemma 2 and $W_x = W/c$, we get $B = \frac{W}{c}k_5\frac{c-\gamma c_{min}}{(1-\gamma)c_{min}}$. Further, since each interface tuned to γc_{min} channels transmit at power level P_0, it transmits data at the rate of $B = \frac{W}{c}k_6$ bits per second. Now turn to the analysis of the upper bound. The difference stems from the need to replace (11) and (12) by different expressions. The expression in (11) and (12) are replaced by the following:

$$H = \sum_{b=1}^{\lambda nT} h(b)$$
$$\leq \left(k_6 \gamma \frac{W}{c} + k_5(1-\gamma)\frac{W}{c}\left(\frac{c-\gamma c_{min}}{(1-\gamma)c_{min}}\right)\right)\frac{nmT}{2} \quad (21)$$

$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} \pi(\frac{\Delta}{2})^2(d_b^h)^2 \leq$$
$$\left(k_6\gamma\frac{W}{c} + k_5(1-\gamma)\frac{W}{c}\left(\frac{c-\gamma c_{min}}{(1-\gamma)c_{min}}\right)\right)Tc_{min} \quad (22)$$

Following the same derivations in section VI, when $c_{min} < c < mn/2$, the capacity is upper bounded by

$$\frac{\sqrt{2}T}{\Delta} \left(k_6\gamma\frac{W}{c} + k_5(1-\gamma)\frac{W}{c}\left(\frac{c-\gamma c_{min}}{(1-\gamma)c_{min}}\right)\right)\sqrt{\frac{nmc_{min}}{\pi}}$$
$$(23)$$

∎

When γ is set to 0 in the equation (23), we get the transport capacity of the arbitrary network under asymmetric allocation as $\frac{\sqrt{2}k_5 WT}{\Delta}\sqrt{\frac{nm}{\pi c_{min}}}$. On the other hand, when γ is set to 1, we get $\frac{\sqrt{2}k_6 T}{\Delta}\frac{W}{c}\sqrt{\frac{nmc_{min}}{\pi}}$ as the transport capacity which indeed is the capacity obtained under power level P_0. Since the maximum network capacity is obtained under $\gamma = 0$ (i.e., when all nodes employ same power $P > P_0$) we can clearly

see that $\frac{\sqrt{2}k_5 WT}{\Delta}\sqrt{\frac{nm}{\pi c_{min}}}$ is indeed the upper bound for the capacity when $c_{min} < c < mn/2$.

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