FRACTAL ANTENNAS

by

Philip Felber

A literature study as a project for ECE 576

Illinois Institute of Technology

December 12, 2000

(Revised: January 16, 2001)

ABSTRACT

Fractal antenna theory is built, as is the case with conventional antenna theory, on classic electromagnetic theory. Fractal antenna theory uses a modern (fractal) geometry that is a *natural* extension of Euclidian geometry. In this report, attention is called to this developing, but already quite large, field of study. In the brief study of fractal antennas for this report, I have found hundreds of papers, dozens of patents, and several established companies manufacturing and selling products. Currently Dwight Jaggard of the University of Pennsylvania and Douglas Werner of Pennsylvania State University and a handful of others are preaching the gospel of fractal antennas.

INTRODUCTION

In the study of antennas, fractal antenna theory is a relatively new area. However, *fractal antennas* and its superset *fractal electrodynamics* [Werner and Mittra, 1999] is a hotbed of research activity. In the research journals, we see reports of active research covering such diverse areas (of *fractal electrodynamics*) as the study of scattering from fractal surfaces (a signature of the surface is imprinted within the scattered field) [Werner and Mittra, 1999], to the study of the radiation from lightening (considering a lightning bolt as a dendrite fractal) [Petrarca, 1999]. I will limit my review to mainly fractal antenna elements and a few words about fractal arrays.

CHRONOLOGY

1947 – Chu and Wheeler establish a fundamental limit on the performance of small antennas [Wheeler, 1947; Chu, 1948].

1983 – Benoit B. Mandelbrot "coins" the word *fractal* (meaning made-up of broken or irregular fragments) [Mandelbrot, 1983].

1986 – Kim and Jaggard report use of fractal arrays in antenna theory [Kim, 1986]. This work reports on using *random fractal arrays*.

1988 - Nathan Cohen builds one of the first known practical fractal antennas in his Boston apartment ham radio station [Cohen, 2000].

1998 - Carles Puente Baliarda of Universitat Politècnica de Catalunya (UPC) wins $230,000 prize for fractal antennas (GSM + DCS single base station antenna), one of three Grand Prizes of The European Information Technology Prize in that year [www.fractus.com, 2000].

BACKGROUND

Talk of chaos and fractals is all around us in the popular media. Here is a snippet from the Michael Crichton science fiction novel *Jurassic Park*.

> "The shorthand is the 'butterfly effect.' A butterfly flaps its wings in Peking, and the weather in New York is different." - Dr. Ian Malcolm explaining chaos theory to Gennaro. "Fractals are a kind of geometry, associated with a man named Mandelbrot." - Malcolm talking to Grant [Crichton, 1990].

The connection between chaos and fractals is that *strange attractors* (the pattern produced by graphing the behavior of a chaotic system) exhibit all the scaling properties associated with fractal objects. Of course, fractals are self-similar objects and possess structure at all scales.

Antenna theory considers three classes of radiators in terms of frequency coverage: (1) narrowband – small range of the order of a few percent around the designed operating frequency, (2) wideband or broadband – covers an octave or two, and (3) frequency independent (a misnomer) – about a ten to one or greater range of frequencies.

Any good antenna text talks about antenna scaling, that is the properties (impedance, efficiency, pattern, etc.) remain the same if all dimensions and the wavelength are scaled by the same factor. Now, remembering that a fractal is a figure that "looks" the same independent of size scaling, we come upon the amazing realization that a fractal shaped metal element can be used as an antenna over a very large band of frequencies. A typical book would say something like, "A distinguishing feature of frequency independent antennas is their self-scaling behavior". But then go on to say, "Frequency independent antennas can be divided into two types: spiral antennas and log-periodic antennas". To remedy this situation a large volume of research has been published on various aspects of fractal antennas and fractal electromagnetics. Just last year (1999) the IEEE published the monumental volume *Frontiers in Electromagnetics* [Werner and Mittra, 1999], which includes several hundred pages on fractal antenna theory.

It is interesting to note that, as fractal geometry is a superset of Euclidian geometry, so is fractal (geometry based) antenna theory a superset of classic (Euclidian geometry) antenna theory. It is somewhat poetic that because of this set to superset relationship, fractal antenna analysis picks up (where classic theory lets off) with the spiral and the log-periodic structures. We are seeing fractal antenna theory shedding new light on our understanding of classic wideband antennas.

Fractals

A fractal is a rough or fragmented geometric shape that can be subdivided in parts, each of which is (at least approximately) a reduced-size copy of the whole. Fractals are generally self-similar and independent of scale. There are many mathematical structures that are fractals; e.g. Sierpinski's gasket, Cantor's comb, von Koch's snowflake, the Mandelbrot set, the Lorenz attractor, et al. Fractals also describe many real-world objects, such as clouds, mountains, turbulence, and coastlines that do not correspond to simple geometric shapes. The terms *fractal* and *fractal dimension* are due to Mandelbrot, who is the person most often associated with the mathematics of fractals [Mandelbrot, 1983]. We can trace the origins of fractal theory, though he did not name it, to Helge von Koch. In 1904 von Koch devised a curve that does not have a tangent anywhere [Lauwerier, 1991]. The curve defined by von Koch is the basis for a class of fractals that bears his name. Also of importance to the early development of fractals (even before they were named by Mandelbrot) is the work of the English meteorologist Lewis Fry Richardson, who studied the relation between the perimeter of an island and the scale of the measurement use to measure it. From Richardson we get the "How long is Britain's west coast?" rhetorical question (published after his death in 1953). All of this work builds on the point-set theory put forth by Georg Cantor (1870) [Lauwerier, 1991].

Mandelbrot included a definition of fractal dimension (of a geometric object) when he first talked about the concept of fractal in 1977 [Lauwerier, 1991]. This definition, based on one given by Hausdorff in 1919, involves a limit process. Basically, it is the change in object size vs. the change in measurement scale, as the measurement scale approaches zero. Logarithms are used for both size and scale.

Antennas

An antenna is any structure or device used to collect or radiate electromagnetic waves. In some sense the first antenna dates from 1887, when Heinrich Hertz designed a brilliant set of wireless experiments to test James Clerk Maxwell's hypothesis. Hertz used a flat dipole for a transmitting antenna and a single turn loop for a receiving antenna. For the next fifty years antenna technology was based on radiating elements configured out of wires and generally supported by wooden poles. These "wire" antennas were the mainstay of the radio pioneers, including: (1) Guglielo Marconi, (2) Edwin Howard Armstrong, and (3) Lee deForest [Lewis, 1991]. Each of these "engineers" has been called the *Father of the Radio* [Lewis, 1991]. Dr. deForest conducted his first long-distance wireless broadcasts, one hundred years ago, from the Illinois Institute of Technology (then it was The Armour Institute) [Fayant, 1907], where he was on the faculty [IIT Technews, 1999]. The first generation antennas were narrowband (small range of the order of a few percent around the designed operating frequency) and were often arrayed to increase directivity. Development of the array culminated in the work of Hidetsu Yagi and Shintaro Uda (1926) [Kraus, 1988]. During this period broadband (see page 4) antennas were also developed. The Yagi-Uda antenna remained king until after the war. Then during the 1950s and 1960s research at the University of Illinois culminated in the development of a class of antennas that became known as frequency independent (see page 4). These have a performance that is periodic in a logarithmic fashion, and have come to be known as "log-periodic" antennas. In recent years, new work (fractal antennas) is coming out of a number of research centers e.g. Boston University, Pennsylvania State University, University of Pennsylvania, et al [Musser, 1999].

Fractal Antennas

As we see fractals have been studied for about a hundred years and antennas have been in use for as long. Fractal antennas are new on the scene. Nathan Cohen, a radio astronomer at Boston University, was a fractal antenna pioneer who experimented with wire fractal antennas (von Koch curves) and fractal planar arrays (Sierpinski triangles). He built the first known fractal antenna in 1988 when he set up a ham radio station at his Boston apartment. Cohen, founder of Fractal Antenna Systems [www.fractenna.com, 2000], is now working with the Amphenol T&M Antennas, located in a Chicago suburb, to make cellular antennas for Motorola cellular phones. In Motorola's application, the fractal arrays have proven to be 25% more efficient than the conventional helical antenna (rubber-ducky) [Musser, 1999].

Much of the manufacturing and research on fractal antennas is being done by Fractal Antenna Systems Inc., a privately-held company with manufacturing facilities in Ft. Lauderdale, Fla., and research and development labs in Belmont, Mass. Also a company in Spain (*Fractus S. A.*) is doing research, registering patents, and marketing fractal antennas. Today the best tools for fractal antenna analysis are *hybrid* type antenna modelers based on traditional Method of Moment (MoM) analysis [Hodges, 1997].

Think of a fractal antenna for what it is: a tuned LC circuit. Remember, the distinction between wideband and multiband is "simply" a question of the size of the gaps between frequencies of similar antenna performance compared to the fineness dictated by the application.

Fifty years ago V. H. Rumsey, a candidate for the title *Father of the Frequency Independent Antenna*, developed what has become our modern notion of broadband antennas. Rumsey's principle is that the impedance and pattern properties of an antenna will be frequency independent if the shape is specified only in terms of angles [Kraus, 1988]. Rumsey's work was, no doubt, inspired by the 1949 publishing of the work of Mushiake and Uda on the constant impedance of self-complementary antennas for all frequencies (half the intrinsic impedance of free space) [ibid].

In our search for the "super" wideband antenna we are driven by two desires: (1) make an antenna for a given frequency band as small as possible, and (2) make an antenna cover several (many) bands. The fractal antenna has performance parameters that repeat periodically with an arbitrary "fineness" dependent on the iteration depth. Therefore, although the finite iteration depth fractal antenna is <u>not</u> frequency independent, it can cover frequency bands arbitrary close together! Also, remembering that radiation comes from accelerating charges, the typical fractal shape (with all those little bends and kinks) makes for good radiation (higher radiation resistance) because of all that acceleration going on as the charges are forced to negotiate all those sharp turns.

Puente carried out early work on fractals as multiband antennas [Puente, 1995], while credit for demonstrating the potential of fractals as small antennas is shared by Puente's group (UPC) and Cohen at the University of Boston [Cohen, 1997].

APPLICATIONS

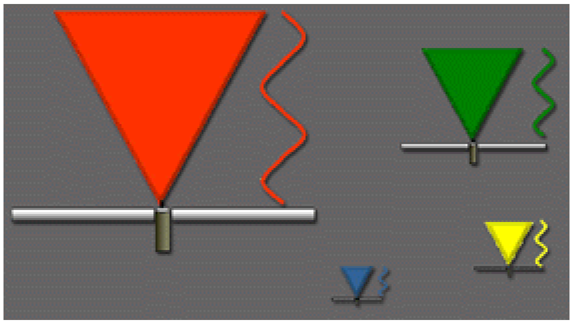
The geometry of the *fractal antenna* encourages its study both as a multiband solution and also as a small (physical size) antenna. First, because one should expect a self-similar antenna (which contains many copies of itself at several scales) to operate in a similar way at several wavelengths. That is, the antenna should keep similar radiation parameters through several bands. Second, because the space-filling properties of some fractal shapes (the fractal dimension) might allow fractal shaped small antennas to better take advantage of the small surrounding space [www.fractus.com, 2000].

Classic wideband antennas

Logarithmic spiral and log-periodic structures can be considered as fractal antennas and can be reanalyzed with fractal antenna theory. Frequency independence in antennas starts with Rumsey's Principle (see page 8). The infinite (logarithmic) spiral is a constant impedance device over all frequencies. We can consider the spiral as "smoothly" self-symmetric. The *log-periodic dipole array* (LPDA) is a wideband device (actually it is multiband with arbitrarily close band spacing). Its various performance properties repeat in the same geometric ratio (log-periodic) as its element size and spacing. We, or course, can consider the LPDA as "discretely" self-symmetric.

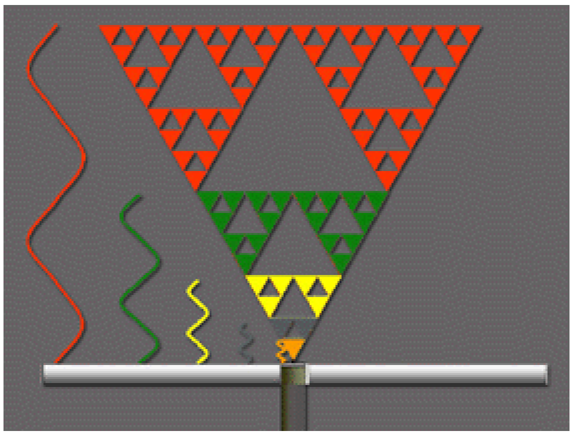
The Sierpinski gasket

The triangular shaped fractal, *Sierpinski gasket,* is named after the Polish mathematician who first described its properties [Puente, April 1998]. With few exceptions (most notably exception being the log-periodic), we typically use a single antenna (size) for each application (frequency band) as depicted in figure 1. This structure, with similarity dimension about 1.58 (log 3 / log 2), is a common study among researchers in fractal antennas [Puente, 2000].



Four antennas (with a wave cartoon) intended to be used on four discrete frequency bands [www.fractus.com, 2000].

Figure 1 Four separate antennas.



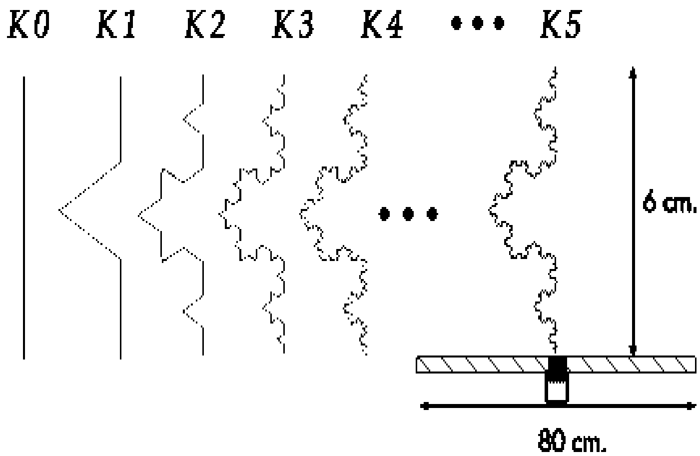
One antenna intended to be use for four discrete frequency bands [www.fractus.com, 2000].

Figure 2 One antenna for four bands.

The von Koch Monopole

In 1998 the von Koch monopole became the first reported fractal small antenna that improved the features of some classical antennas in terms of bandwidth, resonance frequency, and radiation resistance [Puente, January 1998]. Whether such antennas are bound to the fundamental limits on small antennas is still a topic under investigation [Cohen, 1999].

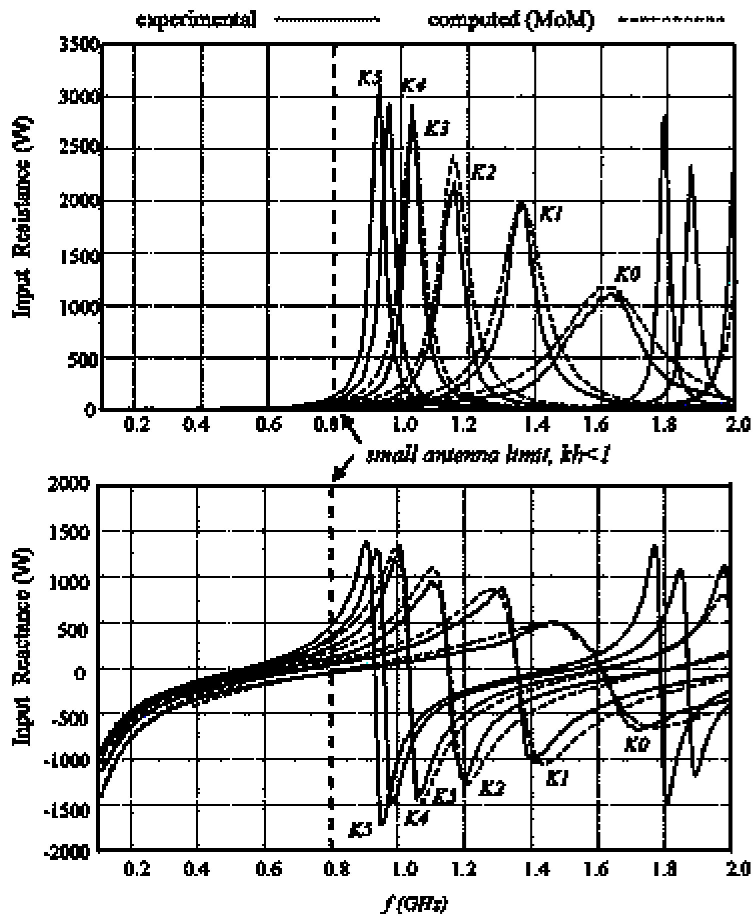
Carles Puente (UPC) reported a study of the von Koch fractal as a monopole antenna [Puente, January 1998]. The von Koch fractal grows by a factor of 4 to 3, giving a fractal similarity dimension of about 1.26 (log 4 / log 3). Puente's team studied the six antennas shown in figure 3.



Von Koch prefractal through five stages of growth [Puente, 1998].

Figure 3 Iterative construction of fractal Koch curve.

They analyzed these antennas by simulation using a MATLAB version of the Moments Method. Figure 4 displays their results as input resistance and reactance for the six antennas. Their analysis shows that the input resistance increases each time the length (not the size) of the antenna is increased. Also, the resonance frequency is shifted towards longer wavelengths becoming resonant antennas even in the small antenna region. A physical explanation of such a behavior might be found in the increasing number, which can be unbounded, of sharp corners and bends of the fractal monopole, which would enhance radiation.



Impedance plots of five prefractal (von Koch) monopole antennas (K1-K5). A straight wire monopole is also included (K0) [Puente, January 1998].

Figure 4 Input resistance and reactance.

When we remember that the von Koch fractal has a similarity dimension of about 1.26 vs. the one-dimensional straight wire, the improvement is not surprising. After all, because of their space filling properties, fractal objects make better use of the available volume inside the radiansphere. Therefore they can be expected to radiate more efficiently than the one-dimensional straight wire [Puente, January 1998].

Fractal Arrays

Started by Kim and Jaggard [Kim, 1986], the study of the fractal random array remains one of the most interesting and fruitful areas in fractal antennas. The fractal random array utilizes a balance of long-range order (typical of fractals) and short-range disorder (typical of random arrays). The randomness provides robustness to element failure while the fractal or periodic structure provides the needed multiband or wideband performance.

Werner and Werner report, from their studies of fractal radiation patterns, that a class of self-similar arrays can be used to synthesize fractal radiation patterns of arbitrary dimension (to be more precise we could say, "similarity dimension") [Werner, 1995]. These arrays are named *Weierstrass arrays* for the mathematician that described this never differentiable but always continuous curve. The Weierstrass "function" is usually developed from an infinite sum of cosines. The Sierpinski carpet, a square version of the previously discussed triangle, has been studied as a deterministic array [Werner, October 1999].

CONCLUSION

Traditional wideband antennas (spiral and log-periodic) and arrays can be analyzed with fractal geometry to shed new light on their operating principles. More to the point, a number of new configurations can be used as antenna elements with good multiband characteristics. Due to the space filling properties of fractals, antennas designed from certain fractal shapes can have far better electrical to physical size ratios than antennas designed from an understanding of shapes in Euclidean space [Werner, 1996]. An incomplete list of fractal shapes that have been used for antennas, monopole and dipole, includes (1) the von Koch curve, (2) the Sierpinski (gasket and carpet) and (3) the fractal tree. Both deterministic and random arrays have been researched, built, and deployed as receiving and transmitting antennas [www.fractenna.com, 2000; www.fractus.com, 2000].

BIBLIOGRAPHY

Chu, "Physical Limitations on omni-directional antennas", *Journal of Applied Physics*, Vol. 19, pp 1163-1175, December 1948.

Cohen, (unsigned) "Antennas, fractals and determination – profile of "Chip" Cohen, N1IR", *WorldRadio* (Online Magazine), November 2000.

Cohen, (Unsigned) "Fractal Antenna White Paper", Fractal Antenna Systems, Inc., 1999.

Cohen, "Fractal Antenna Applications in Wireless Telecommunications", *IEEE Electronics Industries Forum of New England*, pp 43-49, 1997.

Crichton, *Jurassic Park*, Ballantine Books, 1990.

Fayant, "The Wireless Telegraph Bubble", *Success Magazine,* June 1907.

Hodges and Rahmat-Samii, "An Iterative Current-Based Hybrid Method for Complex Structures", *IEEE Transactions on Antennas and Propagation*, Vol. 45, No. 2, pp 265-276, February 1997.

IIT Technews, *The Millennium Project*, 1999.

Kim and Jaggard, "The Fractal Random Array", *Proc. IEEE*, Vol. 74, No. 9, pp 1278-1280, 1986.

Kraus, *Antennas*, Second Edition, McGraw-Hill, 1988.

Lauwerier, *Fractals – Endless Repeated Geometrical Figures*, Princeton University Press, 1991.

Lewis, *Empire of the Air – The Men Who Made Radio*, Harper Collins, 1991.

Mandelbrot, *The Fractal Geometry of Nature*, W. H. Freeman, New York, 1983.

Musser, "Practical Fractals", Wireless Communications, Technology and Business, *Scientific American*, July 1999.

Petrarca, Lupo, Egiziano, Tucci and Vitelli, "Fractal analysis of EM fields generated by tortuous lightning channels", *IEE High Voltage Engineering Symposium*, pp 347-351, 1999.

Puente, Borja, Navarro, and Romeu, "An Iterative Model for Fractal Antennas: Application to the Sierpinski Gasket Antenna", *IEEE Transactions on Antennas and Propagation*, Vol. 48, No. 5, pp 713-719, May 2000.

Puente, Romeu, Pous and Cardama, "On the Behavior of the Sierpinski Multiband Fractal Antenna", *IEEE Transactions on Antennas and Propagation*, Vol. 46, No. 4, pp 517-524, April 1998.

Puente, Romeu, Pous, Ramis and Hijazo, "Small but long Koch fractal monopole", *IEEE Electronics Letters*, Vol. 34, No. 1, pp 9-10, January 1998.

Puente, Pous, Romeu, García, "Antenas Fractales o Multifractales", Invention Patent, n°: P-9501019. Presented at the Oficina Española de Patentes y Marcas. Owner: Universitat Politècnica de Catalunya, 1995.

Werner and Mittra, *Frontiers in Electromagnetics*, IEEE Press Series on Microwave Technology and RF, 1999.

Werner, Haupt and Werner, "Fractal antenna engineering: the theory and design of fractal antenna arrays", *IEEE Antennas and propagation Magazine*, Vol. 41, No. 5, pp 37-58, October 1999.

Werner and Werner, "Frequency-independent features of self-similar fractal antennas", *Radio Science*, Vol. 31, No. 6, pp 1331-1343, November 1996.

Werner and Werner, "On synthesis of fractal radiation patterns", *Radio Science*, Vol. 30, No. 1, pp 29-45, January 1995.

Wheeler, "Fundamental Limitations of Small Antennas", *Proceedings of the IRE*, pp 1479-1488, December 1947.

www.fractenna.com, "WEB site of Fractal Antenna Systems, Inc. in Fort Lauderdale, FL", 2000.

www.fractus.com, "WEB site of Fractus S. A. in Barcelona Spain", 2000.