Exam #1: February 24, 2003

Name: ____________________________________________

Student #: ________________________________________

Instructions: You have 75 minutes to complete the five examination questions that follow. Do all your work on the pages attached to this sheet. Each problem is weighted in the exam grade as shown below. Neatness and organization in your work is important!!!

CLOSED BOOK, CLOSED NOTES, AND NO CALCULATORS ALLOWED!

Some useful formalae and diagrams are provided on the next page.

Grades

1. ________________ (20 pts.)
2. ________________ (20 pts.)
3. ________________ (20 pts.)
4. ________________ (20 pts.)
5. ________________ (20 pts.)

Total ________________ (100 pts.)

Two blank pages are provided at the back for extra work space. If you use these, clearly mark which problems are being worked there.
Some formulae for your use:

\[ t_r \approx \frac{1.8}{\omega_n} \]
\[ t_s = \frac{4.6}{\zeta \omega_n} \quad (1\% \text{ settling time}) \]
\[ M_p = e^{-\pi \zeta / \sqrt{1-\zeta^2}} \]

Laplace transform pairs:

\[ 1(t) \quad \rightarrow \quad \frac{1}{s} \quad \text{(unit step)} \]
\[ e^{-at}1(t) \quad \rightarrow \quad \frac{1}{s + a} \]
\[ te^{-at}1(t) \quad \rightarrow \quad \frac{1}{(s + a)^2} \]
\[ te^{-at}1(t) \quad \rightarrow \quad \frac{1}{(s + a)^2} \]
\[ e^{-at} \cos(bt)1(t) \quad \rightarrow \quad \frac{s + a}{(s + a)^2 + b^2} \]
\[ e^{-at} \sin(bt)1(t) \quad \rightarrow \quad \frac{b}{(s + a)^2 + b^2} \]
\[ \left( 1 - e^{-at} \left( \cos(bt) + \frac{a}{b} \sin(bt) \right) \right)1(t) \quad \rightarrow \quad \frac{a^2 + b^2}{s [(s + a)^2 + b^2]} \]
1. Find the transfer function \( H(s) = Y(s)/R(s) \) for the following system.
2. We wish to control a plant using integral as shown below.

Find the range of values of $K_I$ so that the step response satisfies $t_r < 0.3$ (rise time) and $M_p < 16\%$ (overshoot).
3. Consider the feedback system given by

\[ \frac{1}{(s + 2)(s + 6)} \]

(a) Does the step response exhibit oscillations when \( K = 3 \)? If so, determine the frequency of the oscillations. If not, determine the longest time constant of the system.

(b) Repeat part (a) but with \( K = 13 \).

(c) For what range of \( K \) does the step response of this system exhibit oscillations?
4. The following questions concern the two pairs of complex conjugate poles show in the diagram below.

(a) Which pair corresponds to the smaller overshoot in the step response?

(b) Which corresponds to the smaller rise time?

(c) Which corresponds to the smaller settling time?
5. Consider

\[ H_1(s) = \frac{\left(1 + \frac{s}{\alpha \zeta \omega_n}\right) \omega_n^2}{s^2 + 2\zeta \omega_n + \omega_n^2} \]

\[ H_2(s) = \frac{\omega_n^2}{s^2 + 2\zeta \omega_n + \omega_n^2} \]

\[ H_3(s) = \frac{\omega_n^2}{\left(1 + \frac{s}{\alpha \zeta \omega_n}\right) (s^2 + 2\zeta \omega_n + \omega_n^2)} \]

(a) If \( \zeta = 0.7 \), which of the following best describes conditions on \( \alpha \) so that the step response of \( H_1(s) \) is similar to that of \( H_2(s) \)?

(i) \( \alpha > 1 \)  \hspace{1cm} (ii) \( \alpha > 4 \)  \hspace{1cm} (iii) \( \alpha > 8 \)

(iv) \( \alpha < 1 \)  \hspace{1cm} (v) \( \alpha < 4 \)  \hspace{1cm} (vi) \( \alpha < 8 \)

(b) Which system among \( H_1(s) \), \( H_2(s) \) and \( H_3(s) \) has the largest overshoot in its step response?

(c) Which system among \( H_1(s) \), \( H_2(s) \) and \( H_3(s) \) has the largest rise time in its step response?

(d) Can any of \( H_1(s) \), \( H_2(s) \) and \( H_3(s) \) exhibit a step response \( y(t) \) that is negative for some values of \( t > 0 \)? If so, describe conditions on \( \zeta, \omega_n \) and \( \alpha \) for this to happen.
Extra worksheet.
Extra worksheet.