Examination #2
April 2, 2002

Name: __________________________________________

Student #: ______________________________________

Instructions: The examination lasts for 75 minutes and is closed book, closed notes. Show your work and clearly indicate your final answers on the pages attached to this sheet.

Grades

1. ___________________ (20 pts.)
2. ___________________ (15 pts.)
3. ___________________ (20 pts.)
4. ___________________ (20 pts.)
5. ___________________ (25 pts.)

Total ___________________ (100 pts.)

Some formulae for your use:

\[ t_r \approx \frac{1.8}{\omega_n} \]
\[ t_s = \frac{4.6}{\zeta \omega_n} \quad (1\% \text{ settling time}) \]
\[ M_p = e^{-\pi \zeta / \sqrt{1-\zeta^2}} \]

Laplace transform pairs:

\[ 1(t) \rightarrow \frac{1}{s} \quad \text{(unit step)} \]
\[ e^{-at}1(t) \rightarrow \frac{1}{s + a} \]
\[ te^{-at}1(t) \rightarrow \frac{1}{(s + a)^2} \quad \text{(unit ramp if } a = 0) \]
1. [20 points total] Consider the following feedback system.

First let $D(s) = \frac{4(s + 8)}{(s + 2)}$.

(a) [4 points] What is the steady state error ($\lim_{t\to\infty} e(t)$) when $r(t)$ is a unit step (and $w(t) = 0$)?

(b) [4 points] What is the steady state error when $r(t)$ is a unit ramp (and $w(t) = 0$)?

(c) [4 points] What is the steady state error when $w(t)$ is a unit step (and $r(t) = 0$)?

Now let $D(s) = \frac{4(s + 8)}{s(s + 2)}$ so that the compensator now contains integral action.

(d) [4 points] Repeat part (b).

(e) [4 points] Repeat part (c).
[Extra worksheet for problem 1.]
2. [15 points total]

   (a) [5 points] Write down the transfer function of a PID controller. How many (finite-valued) zeros does it have? How many (finite-valued) poles?

   (b) [5 points] In comparison to proportional control, does PD-control in general increase or decrease the speed of response of the closed loop system? Why?

   (c) [5 points] State one reason for using PI-control.
3. [20 points] Determine the range of $K$ for which

is stable.
4. [20 points] Sketch the root locus showing the closed loop pole positions for

\[
\frac{s + 2}{s(s - 2)(s + 6)}
\]

as \( K \) varies from 0 to \( \infty \). Include in your sketch

- the location of the intersection of the asymptotes (as \( K \to \infty \)) with the real axis (if any),
- the location of \( j\omega \)-axis crossings (if any).

Part of your grade will be determined by the neatness and clarity of your sketch.
[Extra worksheet for problem 4.]
5. **[25 points total]** You wish to design a lead compensator so that

\[ \frac{s + z}{s + p} \frac{6}{s(s + 3)} \]

has a closed loop pole pair at \( s = -3 \pm 3\sqrt{3} \).

(a) **[10 points]** If \( z = 6 \), find values for \( p \) and \( K \) that achieve this objective.

(b) **[10 points]** If \( z = 3 \), find values for \( p \) and \( K \) that achieve this objective.

(c) **[5 points]** Which compensator yields a closed loop with less overshoot in the step response?.
[Extra worksheet for problem 5.]