The examination lasts for 75 minutes and is closed book, closed notes. Below are the examination questions. Show your work and clearly indicate your final answers on the separate set of answer sheets.

1. [15 points] Find the transfer function $H(s) = Y(s)/R(s)$ for the following system.

2. [20 points] Find the two transfer functions $\frac{Y(s)}{R(s)}$ and $\frac{Y(s)}{W(s)}$ in

3. A system $H(s)$ has a pair of complex conjugate poles at $s = -3 \pm j4$.
   (a) [3 points] What is $\omega_n$?
   (b) [3 points] What is $\omega_d$?
   (c) [3 points] What is $\zeta$?
   (d) [3 points] What is $\sigma$?

4. [8 points] Under what conditions on gain $K$ is the response of the system

   $$\frac{K}{s^2 + 2s + K}$$

   considered to be underdamped?
5. Consider the following set of transfer functions.

\[ H_1(s) = \frac{0.5}{s^2+s+0.5} \quad H_2(s) = \frac{1}{s^2+s+1} \]
\[ H_3(s) = \frac{4}{s^2+2s+4} \quad H_4(s) = \frac{8}{s^2+4s+8} \]

(a) [5 points] Which system or systems has a step response with a settling time \( t_s \approx 9.2 \text{sec.} \)?

(b) [5 points] Which system or systems has a step response with a overshoot \( M_p \approx 5\%\)?

6. Consider the following set of transfer functions (the first four of which are the same as in the previous problem.

\[ H_1(s) = \frac{0.5}{s^2+s+0.5} \quad H_2(s) = \frac{1}{s^2+s+1} \]
\[ H_3(s) = \frac{4}{s^2+2s+4} \quad H_4(s) = \frac{8}{s^2+4s+8} \]
\[ H_5(s) = \frac{(8/3)(s+3)}{s^2+2s+4} \quad H_6(s) = \frac{(1/5)(s+5)}{s^2+s+1} \]
\[ H_7(s) = \frac{24}{(s+6)(s^2+2s+4)} \quad H_8(s) = \frac{0.5}{s^2+s+0.5} \]

(a) [5 points] Which system has the smallest rise time in its step response?

(b) [5 points] Which system has the smallest overshoot in its step response?

7. The feedback system below controls the \( y \)-coordinate \( y(t) \) of the robot motion for the robot shown on the following page. The \( x \)-coordinate \( x(t) \) of the motion changes with a constant velocity \( \dot{x}(t) = 2 \text{ m/s} \). With an initial condition of \( x(0) = y(0) = 0 \), we have \( x(t) = 2t \text{ m} \).

At time \( t = 0 \) a step reference command of \( r(t) = 4u_s(t) \) (where \( u_s(t) \) is a unit step) is issued to the control system

\[ \begin{array}{c}
R(s) \\
+ \\
- \\
\frac{K_1}{s(s+2)} \quad Y(s) \\
1 + K_2s
\end{array} \]

that is used to control the robot’s \( y \)-coordinate. The objective is to prevent the robot from striking any of the walls as it enters the corridor to the right.

(a) [10 points] Show that if the step response has no more than a 20\% overshoot and has a 1\% settling time of no more than 3 s., then the requirements are met.

(b) [15 points] Choose \( K_1 \) and \( K_2 \) in the controller so that this happens.
Figure 1: Robot motion diagram.