The following questions are exam questions I have used in the past for the material covered on Exam #2. Although none of these questions asks explicitly about “system type,” that would also be fair game for an exam question.

1. Consider the block diagram shown below.

   (a) Find all values of $K$ such that the closed loop is stable.
   (b) Let $R(s) = 1/s^2$, a unit ramp input. For what values of $K$ is the steady state tracking error less than $3/4$?
   (c) For what values of $K$ does the system track step inputs with zero steady state error?

2. The pole/zero pattern shown below has poles at 0, $-3$, and $-2 \pm j2$, and a zero at $s = -1$.

   (a) Show that the angle of departure from the pole at $s = -2 + j2$ is $-\pi/4$.
   (b) Calculate the intersection of the asymptotes.
   (c) Sketch the root locus.

3. Write down the transfer function of a PI compensator, and explain why it’s called by that name.
4. TRUE/FALSE Answer “true” or “false” to each of the following questions.

(a) If the $G(s)$ has a pole excess of 3 or more (the number of poles minus the number of zeros is at least three), then the closed loop transfer function

$$\frac{KG(s)}{1 + KG(s)}$$

will always be unstable for very large values of positive $K$.

(b) Steady state tracking error in a closed loop system increases as the gain in the loop is increased.

5. Sketch the root locus of the negative unity feedback system whose forward path transfer function is

$$\frac{K(s + 6)}{(s + 4)(s^2 + 4s + 8)}$$

($K$ varies from 0 to $\infty$.) For each point given below, state whether the point lies on (or very close to) the root locus, and if it does, find the value of $K$ which achieves that pole location.

(a) $s = -2$
(b) $s = -1 + j8$
(c) $s = -4 + j8$
(d) $s = -5$

6. Find the steady-state error between the reference and output in

$$\frac{R}{s} + \frac{3s + 2}{s + 4} \rightarrow \frac{1}{s(s + 1)}$$

when $r(t)$ is a unit ramp input.

7. Find the range of $K$ for which the closed loop system

$$\frac{Ks + 20}{s + 2} \rightarrow \frac{s + 10}{s^2}$$

is stable.
8. Draw a neat sketch of the root locus for

\[
K \frac{s + 4}{s(s + 2)^2(s + 5)(s + 6)}
\]

as \(K\) varies between 0 and \(\infty\). You do not need to indicate exact locations of breakaway points or imaginary axis crossings.

9. Consider the feedback control system

\[
\begin{array}{c}
W \\
\downarrow \\
D(s) \\
\downarrow \\
\frac{1}{(s + 2)(s + 4)} \\
\downarrow \\
Y
\end{array}
\]

where \(D(s)\) is a series compensator to be designed. How many poles at \(s = 0\) must \(D(s)\) have if the controlled system is to reject step disturbances \(W(s) = 1/s\) with zero error? Justify your answer.

10. Consider PD control in

\[
\begin{array}{c}
R \\
\downarrow \\
K(1 + T_Ds) \\
\downarrow \\
\frac{1}{s(s + 1)} \\
\downarrow \\
Y
\end{array}
\]

(a) Assuming the proportional control gain \(K\) is held constant, sketch the locus of the closed loop poles as \(T_D\) varies from 0 to \(\infty\). On the diagram, indicate in terms of \(K\) the closed loop poles positions at \(T_D = 0\) (assume \(K > 1/4\)), and indicate in terms of \(K\) the breakaway and breakin point(s).

(b) Using your sketch from (a) as support, give an argument why adding the derivative term in the controller speeds up the closed loop system response.
11. With

\[ G(s) = \frac{s + 3(1 + \sqrt{3})}{s(s + 3)(s^2 + 18s + 90)} \]

\[ G(s) = \frac{s + 3(1 + \sqrt{3})}{s(s + 3)(s^2 + 18s + 90)} \]

\[ \text{in} \]

\[ \begin{array}{c}
R \\
\downarrow
\end{array} \rightarrow
\begin{array}{c}
D(s) \\
\downarrow
\end{array} \rightarrow
\begin{array}{c}
G(s) \\
\uparrow\rightarrow \text{Y}
\end{array} \]

show that it is possible for the compensator

\[ D(s) = K \frac{s + 3 - \sqrt{3}}{s + 3 + \sqrt{3}} \]

to place closed loop poles at \( s = -3 \pm j3 \).

What value of \( K \) achieves these pole locations?

12. What is the steady state tracking error for a unit ramp reference input signal \( R \) in

\[ \begin{array}{c}
R \\
\downarrow
\end{array} \rightarrow
\begin{array}{c}
K \\
\downarrow
\end{array} \rightarrow
\begin{array}{c}
\frac{1}{s(s+5)(s+10)} \\
\uparrow\rightarrow \text{Y}
\end{array} \]

(a) when \( K = 5 \)? (b) when \( K = 1000 \)?

13. For what values of gain \( K \) does the polynomial

\[ s^4 + 6s^3 + 10s^2 + 18s + K \]

have all its roots in the left half plane?
14. For the following three pole/zero plots for $G(s)$, sketch the root locus for $1 + KG(s)$ as $K$ varies from 0 to $+\infty$. Determine in each case all breakaway and break-in points.

(a) The two poles are at $s = -1$ and $s = -3$.

(b) There is a double pole at $s = -1$ and a zero at $s = -3$. 
(c) There are poles at \( s = -4, s = -3 \) and \( s = 2 \) and zeros at \( s = -1 \) and \( s = 0 \).

15. Parts (a), (b), (c) and (d) are in reference to the following feedback control system.

(a) Sketch the root locus for \( 0 < K < \infty \), and specify on your sketch all breakaway points and asymptote intersections.

(b) Show that \( s = j2 \) is on the root locus and determine the value of the gain \( K \) that achieves this closed loop pole.

(c) Determine the steady state tracking error for a unit ramp input when \( K = 12 \) and when \( K = 100 \).

(d) Determine in terms of \( K \) the steady state error from a unit step disturbance (\( w \) is the disturbance). What is the smallest achievable error from a unit step disturbance using the proportional control shown in the diagram?
16. (This problem was originally given as a bonus question.) For

\[ -(+) \longrightarrow K \longrightarrow G(s) \]

assume that

\[ G(s) = \frac{n(s)}{d(s)} \]

and that \( n(s) \) has a root at \( s = 6 \). Show that there is a value \( K_0 \) such that for all \( K > K_0 \) the closed loop system must be unstable.