Sample Questions for Exam #3

The following questions are exam questions I have used in the past for the material covered on Exam #3.

1. With 
   \[ G(s) = \frac{s + 3(1 + \sqrt{3})}{s(s + 3)(s^2 + 18s + 90)} \]
   in
   \[ R \quad + \quad D(s) \quad G(s) \quad Y \]

   show that it is possible for the compensator
   \[ D(s) = K \frac{s + 3 - \sqrt{3}}{s + 3 + \sqrt{3}} \]
   to place closed loop poles at \( s = -3 \pm j3 \).
   What value of \( K \) achieves these pole locations?

2. [10 pts] \( D_1(s) \) has been designed in
   \[ R \quad + \quad D_2(s) \quad D_1(s) \quad G(s) \quad Y \]

   so that the dominant closed loop poles of \( \frac{D_1G}{1 + D_1G} \) are at \( s = -5 \pm j3 \). \( K_v \) for this closed loop is determined to be 10. Design \( D_2(s) \) to increase \( K_v \) to 200 without influencing the position of the dominant closed loop poles.

3. Consider the feedback control system
   \[ R \quad + \quad D(s) \quad \frac{1}{(s+2)(s+4)} \quad Y \]
where $D(s)$ is a series compensator to be designed.

(a) Design $D(s)$ as the proportional controller that minimizes the tracking error of a step reference input while keeping the overshoot in the step response less than 5% ($\zeta \geq 0.707$).

(b) How many poles at $s = 0$ must $D(s)$ have if the controlled system is to reject step disturbances $W(s) = 1/s$ with zero error? Justify your answer.

(c) Design $D(s)$ as a PI controller that achieves a closed loop pole pair at $s = -2 \pm j2$.

4. The open loop bode plot of $G(s)$ is

Mark on the Bode plot how to measure (a) gain margin, (b) phase margin, (c) crossover frequency. Estimate each quantity from the plot.

5. Consider PD control in

(a) Assuming the proportional control gain $K$ is held constant, sketch the locus of the closed loop poles as $T_D$ varies from 0 to $\infty$. On the diagram, indicate in terms
of $K$ the closed loop poles positions at $T_D = 0$ (assume $K > 1/4$), and indicate in terms of $K$ the breakaway and breakin point(s).

(b) Using your sketch from (a) as support, give an argument why adding the derivative term in the controller speeds up the closed loop system response.

6. It is desired to choose $C(s)$ in

\[
\begin{align*}
\begin{array}{c}
+ \\
- \\
\end{array} & \rightarrow \\
 & C(s) \\
& \rightarrow \frac{1}{s^2} \\
\end{align*}
\]

so that the dominant closed loop pole pair is at $s = -1 \pm j\sqrt{3}$. If the controller is given by

\[ C(s) = K \frac{s + 1}{s + a}. \]

find $a$ and $K$ to meet the desired objectives.

7. It is desired to design $C(s)$ in

\[
\begin{align*}
\begin{array}{c}
+ \\
- \\
\end{array} & \rightarrow \\
 & C(s) \\
& \rightarrow \frac{1}{s^2 + 25} \\
\end{align*}
\]

to control the highly resonant plant $G(s) = \frac{1}{s^2 + 25}$. Two candidates for the controller transfer function are

(I.) $C(s) = K \frac{s^2 + 16}{s(s + 5)}$, and (II.) $C(s) = K \frac{s^2 + 36}{s(s + 5)}$

Which of the controllers is able to provide a more satisfactory response? Justify your answer.

8. In the diagram

\[
\begin{align*}
\begin{array}{c}
+ \\
- \\
\end{array} & \rightarrow \\
 & C(s) \\
& \rightarrow \frac{10}{s + 10} \\
\end{align*}
\]

The controller $C(s)$ is to be designed so that

- the dominant poles all lie to the left of Re($s$) = 4, and all have damping ratio $\zeta < 1/2$ (or $\theta < \pi/3$)
• the closed loop is type 1 with $K_v \geq 250$.

Determine a controller which meets these objectives.

9. TRUE/FALSE Answer “true” or “false” to each of the following questions.

   (a) The type of a negative unity feedback control system is equal to the number of zeros at $s = 0$ in the forward path.
   (b) In a lag compensator with transfer function

   $C(s) = \frac{s + z}{s + p},$

   we have $z < p$.
   (c) A lead compensator tends to enhance the stability of the closed loop system.

10. The Nyquist plot of a transfer function $G(s)$ is given in your exam booklet. $G(s)$ is known to have 2 poles in the right half plane. For what values of positive $K$ is

   $\frac{KG(s)}{1 + KG(s)}$

stable? Be sure to indicate how you found your answer.

11. Answer the following questions with regard to the Bode plots included in the exam booklet. $G(s)$ refers to the transfer function whose Bode plots are given.

   (a) What is the crossover frequency of $G(s)$? Mark it clearly on the Bode plots.
   (b) Give the phase margin of

   $\frac{10G(s)}{1 + 10G(s)}$.

   Clearly indicate on the plots how you determined this value.