The examination lasts for 75 minutes and is closed book, closed notes. No electronic devices are permitted, including but not limited to calculators, cellphones, and other handheld devices. (Any such items in the examination room must be off and put away, subject to a 20 point penalty for the first violation and a score of 0 on the exam for the second violation.)

Do all your work on the pages in this exam booklet. Do not un staple these pages. Any un stapled or restapled pages will NOT be graded. You may write on the backs of the pages if you need to. Each of problems 4 and 5 have an extra work page immediately following them, and attached at the back of the exam booklet is one more extra work page. (Clearly mark the problem number associated with your work if you use this last page.)

Show your work and clearly indicate your final answers. Neatness and organization in your work is important and will influence your grade.

Each problem is weighted toward the final total as shown below.

Grades

1. _______________ (12 pts.)
2. _______________ (10 pts.)
3. _______________ (18 pts.)
4. _______________ (20 pts.)
5. _______________ (20 pts.)

Total _______________ (80 pts.)
1. [18 points] For the signal $x(t)$ shown below, express $x(t)$ as a linear combination of shifted steps and ramps.

\[
x(t) = -r(t+1) + 3r(t-1) - 3r(t-3) + r(t-4) - r(t-6) + r(t-7)
\]
2. [10 points] Sketch the graph of \( x(t) \) as a function of \( t \) for \(-5 \leq t \leq 8\) where

\[
x(t) = 3u(t + 2) - 5u(t) + 2r(t - 1) - 2r(t - 3) - 2u(t - 5).
\]
3. [18 points; 3 points each] Mark the following statements as either TRUE or FALSE by circling the appropriate response.

(a) The system
\[ y(t) = \int_{-\infty}^{t} x(\lambda) \, d\lambda \]
has memory.

\[ \text{TRUE} \quad \text{FALSE} \]

(b) The system
\[ y(t) = \int_{0}^{+\infty} e^{-\lambda} x(t - \lambda) \, d\lambda \]
is the same as the system
\[ y(t) = \int_{-\infty}^{+\infty} e^{-\lambda} u(\lambda) x(t - \lambda) \, d\lambda \]

\[ \text{TRUE} \quad \text{FALSE} \]

(c) The system
\[ y(t) = \int_{-\infty}^{+\infty} e^{-(t-\lambda)} u(t - \lambda) x(\lambda) \, d\lambda \]
is the same as the system
\[ y(t) = \int_{-\infty}^{+\infty} e^{-\lambda} u(\lambda) x(t - \lambda) \, d\lambda \]

\[ \text{TRUE} \quad \text{FALSE} \]
(d) If a system is linear and time-invariant and the input
\[ x_1(t) = e^{-t}u(t) \]
produces the output
\[ y_1(t) = (1 - e^{-t}) u(t), \]
then when the input is
\[ x_2(t) = e^{-t}u(t - 2) \]
the output is
\[ y_2(t) = e^{-2}(1 - e^{-(t-2)}) u(t - 2). \]

TRUE   FALSE

(e) The system
\[ y(t) = h(t) * x(t) \]
is linear and time-invariant when
\[ h(t) = e^{-|t|}(u(t+1) - u(t-1)). \]

TRUE   FALSE

(f) The system
\[ y(t) = h(t) * x(t) \]
is noncausal when
\[ h(t) = e^{-(t-2)}u(t - 2). \]

TRUE   FALSE
4. [20 points] Determine the convolution \( y(t) = h(t) \ast x(t) \) and sketch the plot of \( y(t) \) when \( h(t) \) and \( x(t) \) are as shown below.

\[ h(t) \]

\[ x(t) \]

\[ y(t) \]
EXTRA WORKSHEET for problem 4
5. [20 points] Find \( y(t) = h(t) \ast x(t) \) when

(a) \[
\begin{align*}
h(t) &= 3e^{-(t-2)}u(t-2) \\
x(t) &= u(t+2)
\end{align*}
\]

(b) \[
\begin{align*}
h(t) &= e^{-(t+1)}u(t+1) \\
x(t) &= 3u(t-3)
\end{align*}
\]

(c) \[
\begin{align*}
h(t) &= \sqrt{3}e^{-t}u(t) \\
x(t) &= \sqrt{3}(u(t) - u(t-2))
\end{align*}
\]

Hint: what is \( y(t) \) when \( h(t) = e^{-t}u(t) \) and \( x(t) = u(t) \)?

\[
e^{-t}u(t) \ast u(t) = (1-e^{-t})u(t)
\]

(a) \( y(t) = 3(1-e^{-t})u(t) \)

(b) \( y(t) = 3(1-e^{-(t-2)})u(t-2) \)

(c) \[
\begin{align*}
y(t) &= 3(1-e^{-t})u(t) \\
&\quad - 3(1-e^{-(t-2)})u(t-2)
\end{align*}
\]
EXTRA WORKSHEET for problem 5
Extra worksheet.