Name: __________________________________________

Instructions:

The examination lasts for 120 minutes and is closed book, closed notes. Calculators are okay to use, but no other electronic devices are permitted, including but not limited to cellphones, and other handheld devices. (Any such items in the examination room must be off and put away, subject to a 20 point penalty for the first violation and a score of 0 on the exam for the second violation.)

A set of tables of properties of various transforms, common transform pairs, and some other information is provided for your convenience.

There are five problems on the exam.

Do all your work on the pages in this exam booklet. Do not un staple these pages. Any un stapled or restapled pages will NOT be graded. There is an extra worksheet following each problem page, and attached at the back of the exam booklet is one more extra work page. You may write on the backs of the pages if you need to.

Show your work and clearly indicate your final answers. Neatness and organization in your work is important and will influence your grade.

Each problem is weighted toward the final total as shown below.

Grades

1. ____________________ (20 pts.)
2. ____________________ (20 pts.)
3. ____________________ (20 pts.)
4. ____________________ (20 pts.)
5. ____________________ (20 pts.)
Total ____________________ (100 pts.)
### Some properties of the Fourier transform

<table>
<thead>
<tr>
<th>Property</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linearity</td>
<td>$a x(t) + b v(t) \leftrightarrow a X(\omega) + b V(\omega)$</td>
</tr>
<tr>
<td>Time shift</td>
<td>$x(t - c) \leftrightarrow X(\omega)e^{-j\omega c}$</td>
</tr>
<tr>
<td>Time scaling</td>
<td>$x(at) \leftrightarrow \frac{1}{a} X\left(\frac{\omega}{a}\right)$, for $a &gt; 0$</td>
</tr>
<tr>
<td>Time reversal</td>
<td>$x(-t) \leftrightarrow X(-\omega)$</td>
</tr>
<tr>
<td>Multiplication by a power of $t$</td>
<td>$t^n x(t) \leftrightarrow j^n \frac{d^n}{d\omega^n} X(\omega)$, $n = 1, 2, \ldots$</td>
</tr>
<tr>
<td>Multiplication by sinusoids</td>
<td>$x(t) e^{j\omega_0 t} \leftrightarrow X(\omega - \omega_0)$, for $\omega_0$ real</td>
</tr>
<tr>
<td></td>
<td>$x(t) \cos(\omega_0 t) \leftrightarrow \frac{1}{2} [X(\omega + \omega_0) + X(\omega - \omega_0)]$</td>
</tr>
<tr>
<td></td>
<td>$x(t) \sin(\omega_0 t) \leftrightarrow \frac{1}{2} [X(\omega + \omega_0) - X(\omega - \omega_0)]$</td>
</tr>
<tr>
<td>Differentiation</td>
<td>$\frac{d^n}{dt^n} x(t) \leftrightarrow (j\omega)^n X(\omega)$, $n = 1, 2, \ldots$</td>
</tr>
<tr>
<td>Integration</td>
<td>$\int_{-\infty}^{t} x(\lambda) d\lambda \leftrightarrow \frac{1}{j\omega} X(\omega) + \pi X(0) \delta(\omega)$</td>
</tr>
<tr>
<td>Convolution</td>
<td>$x(t) \ast v(t) \leftrightarrow X(\omega) V(\omega)$</td>
</tr>
<tr>
<td>Multiplication</td>
<td>$x(t)v(t) \leftrightarrow \frac{1}{2\pi} X(\omega) \ast V(\omega)$</td>
</tr>
<tr>
<td>Parseval’s theorem</td>
<td>$\int_{-\infty}^{\infty} x(t)v(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) V(\omega) d\omega$</td>
</tr>
<tr>
<td>Parseval’s, special case</td>
<td>$\int_{-\infty}^{\infty} x^2(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty}</td>
</tr>
<tr>
<td>Duality</td>
<td>$X(t) \leftrightarrow 2\pi x(-\omega)$</td>
</tr>
</tbody>
</table>
Some Fourier transform pairs

\[ 1, \ -\infty < t < \infty \quad \longleftrightarrow \quad 2\pi \delta(\omega) \]

\[ -0.5 + u(t) \quad \longleftrightarrow \quad \frac{1}{j \omega} \]

\[ u(t) \quad \longleftrightarrow \quad \frac{1}{j \omega} + \pi \delta(\omega) \]

\[ \delta(t) \quad \longleftrightarrow \quad 1 \]

\[ \delta(t - c) \quad \longleftrightarrow \quad e^{-j\omega c}, \ c \text{ any real number} \]

\[ e^{-at} u(t), \ a > 0 \quad \longleftrightarrow \quad \frac{1}{j \omega + a} \]

\[ e^{j\omega_0 t} \quad \longleftrightarrow \quad 2\pi \delta(\omega - \omega_0) \]

\[ p_\tau(t) = \begin{cases} 
1, & -\tau/2 < t < \tau/2 \\
0, & \text{otherwise} 
\end{cases} \quad \longleftrightarrow \quad \tau \ \text{sinc} \left( \frac{\tau \omega}{2\pi} \right) \]

\[ \tau \ \text{sinc} \left( \frac{\tau t}{2\pi} \right) \quad \longleftrightarrow \quad 2\pi p_\tau(\omega) \]

\[ \left( 1 - \frac{2|t|}{\tau} \right) p_\tau(t) \quad \longleftrightarrow \quad \frac{\tau}{2} \ \text{sinc}^2 \left( \frac{\tau \omega}{4\pi} \right) \]

\[ \frac{\tau}{2} \ \text{sinc}^2 \left( \frac{\tau \omega}{4\pi} \right) \quad \longleftrightarrow \quad 2\pi \left( 1 - \frac{2|\omega|}{\tau} \right) p_\tau(\omega) \]

\[ \cos(\omega_0 t) \quad \longleftrightarrow \quad \pi \left[ \delta(\omega + \omega_0) + \delta(\omega - \omega_0) \right] \]

\[ \sin(\omega_0 t) \quad \longleftrightarrow \quad j\pi \left[ \delta(\omega + \omega_0) - \delta(\omega - \omega_0) \right] \]
Some properties of the discrete-time Fourier transform

<table>
<thead>
<tr>
<th>Linearity</th>
<th>$ax[n] + bv[n] \longleftrightarrow aX(\Omega) + bV(\Omega)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time shift</td>
<td>$x[n - q] \longleftrightarrow X(\Omega)e^{-jq\Omega}$, $q$ any integer</td>
</tr>
<tr>
<td>Time reversal</td>
<td>$x[-n] \longleftrightarrow X(-\Omega) = \overline{X}(\Omega)$</td>
</tr>
<tr>
<td>Multiplication by $n$</td>
<td>$nx[n] \longleftrightarrow j \frac{d}{d\Omega}X(\Omega)$</td>
</tr>
<tr>
<td>Multiplication by $e^{j\Omega_0n}$</td>
<td>$x[n]e^{j\Omega_0n} \longleftrightarrow X(\Omega - \Omega_0)$, for $\Omega_0$ real</td>
</tr>
<tr>
<td>Multiplication by $\sin(\Omega_0n)$</td>
<td>$x[n]\sin(\Omega_0n) \longleftrightarrow \frac{1}{2} [X(\Omega + \Omega_0) - X(\Omega - \Omega_0)]$</td>
</tr>
<tr>
<td>Multiplication by $\cos(\Omega_0n)$</td>
<td>$x[n]\cos(\Omega_0n) \longleftrightarrow \frac{1}{2} [X(\Omega + \Omega_0) + X(\Omega - \Omega_0)]$</td>
</tr>
<tr>
<td>Convolution</td>
<td>$x[n] * v[n] \longleftrightarrow X(\Omega)V(\Omega)$</td>
</tr>
<tr>
<td>Multiplication</td>
<td>$x[n]v[n] \longleftrightarrow \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\Omega - \lambda)V(\lambda)d\lambda$</td>
</tr>
<tr>
<td>Parseval’s theorem</td>
<td>$\sum_{n=-\infty}^{\infty} x[n]v[n] \longleftrightarrow \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\Omega)V(\Omega)d\Omega$</td>
</tr>
<tr>
<td>Special case of Parseval’s theorem</td>
<td>$\sum_{n=-\infty}^{\infty} x^2[n] \longleftrightarrow \frac{1}{2\pi} \int_{-\pi}^{\pi}</td>
</tr>
</tbody>
</table>

Some discrete-time Fourier transform pairs

<table>
<thead>
<tr>
<th>$x[n]$</th>
<th>$X(\Omega)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta[n]$</td>
<td>1</td>
</tr>
<tr>
<td>$u[n]$</td>
<td>$\frac{1}{1 - e^{-j\Omega}} + \sum_{k=-\infty}^{\infty} \pi \delta(\Omega - 2\pi k)$</td>
</tr>
<tr>
<td>$\text{for } a^n u[n] \text{ with }</td>
<td>a</td>
</tr>
<tr>
<td>$x[n] = \begin{cases} 1, &amp; n = -q, \ldots, -1, 0, 1, \ldots, q \ 0, &amp; \text{all other } n \end{cases}$</td>
<td>$X(\Omega) = \frac{\sin[(q+\frac{1}{2})\Omega]}{\sin(\Omega/2)}$</td>
</tr>
<tr>
<td>$x[n] = \frac{B}{\pi} \text{sinc} \left( \frac{B}{\pi} n \right)$</td>
<td>$X(\Omega) = \begin{cases} 1, &amp;</td>
</tr>
</tbody>
</table>
Some properties of the Laplace transform

<table>
<thead>
<tr>
<th>Property</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linearity</td>
<td>$ax(t) + bv(t) \leftrightarrow aX(s) + bV(s)$</td>
</tr>
<tr>
<td>Time shift [single-sided]</td>
<td>$x(t - c)u(t - c) \leftrightarrow X(s)e^{-cs}$ for $c &gt; 0$</td>
</tr>
<tr>
<td>Time shift [double-sided]</td>
<td>$x(t - c) \leftrightarrow X(s)e^{-cs}$</td>
</tr>
<tr>
<td>Time scaling</td>
<td>$x(at) \leftrightarrow \frac{1}{a}X\left(\frac{s}{a}\right)$, for $a &gt; 0$</td>
</tr>
<tr>
<td>Multiplication by a power of $t$</td>
<td>$t^Nx(t) \leftrightarrow (-1)^N \frac{d^N}{ds^N}X(s)$, $N = 1, 2, \ldots$</td>
</tr>
<tr>
<td>Multiplication by exponentials</td>
<td>$x(t)e^{at} \leftrightarrow X(s - a)$, $a$ real or complex</td>
</tr>
<tr>
<td>$x(t)\cos(\omega_0 t) \leftrightarrow \frac{1}{2} \left[ X(s + j\omega_0) + X(s - j\omega_0) \right]$</td>
<td></td>
</tr>
<tr>
<td>$x(t)\sin(\omega_0 t) \leftrightarrow \frac{j}{2} \left[ X(s + j\omega_0) - X(s - j\omega_0) \right]$</td>
<td></td>
</tr>
<tr>
<td>Differentiation [single-sided]</td>
<td>$\frac{d}{dt}x(t) \leftrightarrow sX(s) - x(0)$</td>
</tr>
<tr>
<td>Differentiation [double-sided]</td>
<td>$\frac{d}{dt}x(t) \leftrightarrow sX(s)$</td>
</tr>
<tr>
<td>Integration [single-sided]</td>
<td>$\int_0^t x(\lambda)d\lambda \leftrightarrow \frac{1}{s}X(s)$</td>
</tr>
<tr>
<td>Integration [double-sided]</td>
<td>$\int_{-\infty}^t x(\lambda)d\lambda \leftrightarrow \frac{1}{s}X(s)$</td>
</tr>
<tr>
<td>Convolution</td>
<td>$x(t) \ast v(t) \leftrightarrow X(s)V(s)$</td>
</tr>
<tr>
<td>Initial-value theorem</td>
<td>$x(0) = \lim_{s \to \infty} sX(s)$</td>
</tr>
<tr>
<td>$\dot{x}(0) = \lim_{s \to \infty} [s^2X(s) - sx(0)]$</td>
<td></td>
</tr>
<tr>
<td>$x^{(N)}(0) = \lim_{s \to \infty} \left[ s^{N+1}X(s) - s^N x(0) - s^N \dot{x}(0) - \ldots - sX^{(N-1)}(0) \right]$</td>
<td></td>
</tr>
<tr>
<td>Final-value theorem</td>
<td>If $\lim_{t \to \infty} x(t)$ exists, then $\lim_{t \to \infty} x(t) = \lim_{s \to 0} sX(s)$</td>
</tr>
</tbody>
</table>
Some Laplace transform pairs

\[ \delta(t) \leftrightarrow 1 \]
\[ u(t) \leftrightarrow \frac{1}{s} \]
\[ u(t) - u(t - c) \leftrightarrow \frac{1 - e^{-cs}}{s}, \quad c > 0 \]
\[ t^N u(t) \leftrightarrow \frac{N!}{s^{N+1}} \]
\[ e^{-at}u(t), \quad a > 0 \leftrightarrow \frac{1}{s + a}, \quad a \text{ real or complex} \]
\[ t^N e^{-at}u(t), \quad a > 0 \leftrightarrow \frac{N!}{(s + a)^{N+1}} \]
\[ \cos(\omega_0 t)u(t) \leftrightarrow \frac{s}{s^2 + \omega_0^2} \]
\[ \sin(\omega_0 t)u(t) \leftrightarrow \frac{\omega_0}{s^2 + \omega_0^2} \]
\[ e^{-bt} \cos(\omega_0 t)u(t) \leftrightarrow \frac{(s + b)}{(s + b)^2 + \omega_0^2} \]
\[ e^{-bt} \sin(\omega_0 t)u(t) \leftrightarrow \frac{\omega_0}{(s + b)^2 + \omega_0^2} \]
The sinc function is defined by
\[ \text{sinc}(x) = \frac{\sin(\pi x)}{\pi x} \]

\[
\sum_{k=n_0}^{n_1} a^n = \frac{a^{n_0} - a^{n_1+1}}{1 - a}, \quad a \neq 1
\]

\[
\sum_{k=n_0}^{\infty} a^n = \frac{a^{n_0}}{1 - a}, \quad |a| < 1
\]

\[
\frac{1 - e^{-j2N\omega}}{1 - e^{-j2M\omega}} = e^{-jN\omega}(e^{jN\omega} - e^{-jN\omega}) = e^{-j(N-M)\omega} \frac{\sin(N\omega)}{\sin(M\omega)}
\]

Some trigonometric identities

\[ \sin(\alpha \pm \beta) = \sin(\alpha) \cos(\beta) \pm \cos(\alpha) \sin(\beta) \]
\[ \cos(\alpha \pm \beta) = \cos(\alpha) \cos(\beta) \mp \sin(\alpha) \sin(\beta) \]
\[ \sin(\alpha) \cos(\beta) = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)] \]
\[ \cos(\alpha) \cos(\beta) = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)] \]
\[ \sin(\alpha) \sin(\beta) = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)] \]

Common trigonometric function values

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( \cos(\theta) )</th>
<th>( \sin(\theta) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>( \pi/6 )</td>
<td>( \sqrt{3}/2 )</td>
<td>1/2</td>
</tr>
<tr>
<td>( \pi/4 )</td>
<td>( \sqrt{2}/2 )</td>
<td>( \sqrt{2}/2 )</td>
</tr>
<tr>
<td>( \pi/3 )</td>
<td>1/2</td>
<td>( \sqrt{3}/2 )</td>
</tr>
</tbody>
</table>
1. **[20 points]** A continuous-time LTI system

![Diagram showing a continuous-time LTI system](image)

has frequency response

\[ H(\omega) = \frac{25 - \omega^2}{(2 + j\omega)^2}. \]

Find \( y(t) \) if

\[ x(t) = 4 + 8 \cos(2t + \pi/4) - 7 \cos(5t - \pi/6). \]
Extra worksheet for problem 1
2. [20 points] The signal

\[ x[n] = \sqrt{2} \cos \left( \frac{\pi}{6} n + \frac{\pi}{4} \right) \quad (\text{for } -\infty < n < \infty) \]

is applied to a discrete-time LTI system with unit-pulse response

\[ h[n] = \left( \frac{2}{1 + \sqrt{3}} \right)^n u[n]. \]

What is the output \( y[n] \) for \(-\infty < n < \infty\)?
Extra worksheet for problem 2
3. [20 points] The periodic signal $x(t)$ shown below is applied as the input of an LTI system with frequency response

$$H(\omega) = \begin{cases} \frac{\pi}{3}, & |\omega| \leq 1.5 \\ 0, & \text{otherwise} \end{cases}$$

Determine the output $y(t)$ of the LTI system.
Extra worksheet for problem 3
4. [20 points] If
\[ x(t) = 4 \left[ e^{-3t}u(t) - e^{-3(t-4)}u(t-4) \right], \quad v(t) = 3e^{-4t}u(t), \]
find \( y(t) = x(t) * v(t) \).
Extra worksheet for problem 4
5. [20 points] The feedback control of a DC motor is implemented so that the rotational position of the motor shaft $y(t)$, in terms of a reference voltage $x(t)$ applied to the motor windings, is described by the differential equation

$$\frac{d^2 y(t)}{dt^2} + 11 \frac{dy(t)}{dt} + 30 y(t) = 30 x(t).$$

If $x(t) = u(t)$, find $y(t)$. 
Extra worksheet for problem 5
Extra worksheet