Name: ____________________________________________

Instructions:

The examination lasts for 75 minutes and is closed book, closed notes. No electronic devices are permitted, including but not limited to calculators, cellphones, and other handheld devices. (Any such items in the examination room must be off and put away, subject to a 20 point penalty for the first violation and a score of 0 on the exam for the second violation.)

A table of properties of the Fourier transform is attached for your convenience, as is a brief table of Fourier transform pairs, and also a couple of trigonometric identities. There are five problems on the exam.

Do all your work on the pages in this exam booklet. Do not unstaple these pages. Any unstapled or restapled pages will NOT be graded. You may write on the backs of the pages if you need to, and attached at the back of the exam booklet are two extra work pages.

Show your work and clearly indicate your final answers. Neatness and organization in your work is important and will influence your grade.

Each problem is weighted toward the final total as shown below.

Grades

1. ____________________ (20 pts.)
2. ____________________ (20 pts.)
3. ____________________ (20 pts.)
4. ____________________ (20 pts.)
5. ____________________ (20 pts.)
Total ________________ (100 pts.)
<table>
<thead>
<tr>
<th>Property</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linearity</td>
<td>( ax(t) + bv(t) \leftrightarrow aX(\omega) + bX(\omega) )</td>
</tr>
<tr>
<td>Time shift</td>
<td>( x(t - c) \leftrightarrow X(\omega)e^{-j\omega c} )</td>
</tr>
<tr>
<td>Time scaling</td>
<td>( x(at) \leftrightarrow \frac{1}{a} X\left(\frac{\omega}{a}\right), \text{ for } a &gt; 0 )</td>
</tr>
<tr>
<td>Time reversal</td>
<td>( x(-t) \leftrightarrow X(-\omega) )</td>
</tr>
<tr>
<td>Multiplication by a power of t</td>
<td>( t^n x(t) \leftrightarrow j^n \frac{d^n}{d\omega^n} X(\omega), \text{ for } n = 1, 2, \ldots )</td>
</tr>
<tr>
<td>Multiplication by sinusoids</td>
<td>( x(t) e^{j\omega_0 t} \leftrightarrow X(\omega - \omega_0), \text{ for } \omega_0 \text{ real} )</td>
</tr>
<tr>
<td>Multiplication by sinusoids</td>
<td>( x(t) \cos(\omega_0 t) \leftrightarrow \frac{1}{2} [X(\omega + \omega_0) + X(\omega - \omega_0)] )</td>
</tr>
<tr>
<td>Multiplication by sinusoids</td>
<td>( x(t) \sin(\omega_0 t) \leftrightarrow \frac{j}{2} [X(\omega + \omega_0) - X(\omega - \omega_0)] )</td>
</tr>
<tr>
<td>Differentiation</td>
<td>( \frac{d^n}{dt^n} x(t) \leftrightarrow (j\omega)^n X(\omega), \text{ for } n = 1, 2, \ldots )</td>
</tr>
<tr>
<td>Integration</td>
<td>( \int_{-\infty}^{t} x(\lambda)d\lambda \leftrightarrow \frac{1}{j\omega} X(\omega) + \pi X(0)\delta(\omega) )</td>
</tr>
<tr>
<td>Convolution</td>
<td>( x(t) \ast v(t) \leftrightarrow X(\omega)V(\omega) )</td>
</tr>
<tr>
<td>Multiplication</td>
<td>( x(t)v(t) \leftrightarrow \frac{1}{2\pi} X(\omega) \ast V(\omega) )</td>
</tr>
<tr>
<td>Duality</td>
<td>( X(t) \leftrightarrow 2\pi x(-\omega) )</td>
</tr>
</tbody>
</table>
Some Fourier transform pairs

\[ \delta(t) \iff 1 \]
\[ u(t) \iff \frac{1}{j\omega + \pi \delta(\omega)} \]
\[ e^{-at}u(t), \ a > 0 \iff \frac{1}{j\omega + a} \]
\[ p_T(t) \iff T \text{sinc} \left( \frac{T\omega}{2\pi} \right) \]

Function definitions

\[ p_T(t) = \begin{cases} 
1, & -T/2 < t < T/2 \\
0, & \text{otherwise} 
\end{cases} \]

\[ \text{sinc}(x) = \frac{\sin(\pi x)}{\pi x} \]

Some trigonometric identities

\[ \sin(\alpha \pm \beta) = \sin(\alpha) \cos(\beta) \pm \cos(\alpha) \sin(\beta) \]
\[ \cos(\alpha \pm \beta) = \cos(\alpha) \cos(\beta) \mp \sin(\alpha) \sin(\beta) \]
\[ \sin(\alpha) \cos(\beta) = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)] \]
\[ \cos(\alpha) \cos(\beta) = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)] \]
\[ \sin(\alpha) \sin(\beta) = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)] \]
1. [20 points total] The trigonometric Fourier series expansion for $x(t)$ given by

$$x(t) = \sum_{k=1}^{\infty} \frac{2}{k\pi} \left( 1 - \cos \left( \frac{k\pi}{3} \right) \right) \sin(k\pi t).$$

(a) [10 points] Find the exponential Fourier series for $x(t)$.

(b) [10 points] Find the trigonometric Fourier series expansion for $y(t)$ given by
2. [20 points total]

(a) [10 points] Find the Fourier transform $X(\omega)$ of

$$x(t) = e^{-\frac{1}{2}t} \ u(t - 2).$$

(b) [10 points] Find $y(t)$ if its Fourier transform $Y(\omega)$ is

$$Y(\omega) = e^{-\frac{1}{4}(\omega-2)} \ u(\omega).$$
3. [20 points total]

(a) [10 points] Show that if
\[ X(\omega) = 2\pi\delta(\omega) \]
then \( x(t) = 1 \).

(b) [10 points] If
\[ X(\omega) = 3\pi\left( \frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2} \right)\delta(\omega - 6) + 3\pi\left( \frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2} \right)\delta(\omega + 6), \]
what is \( x(t) \)?
4. [20 points total] Suppose that

\[ X(\omega) = \begin{cases} 
4 - \omega^2, & |\omega| < 2 \\
0, & \text{otherwise.}
\end{cases} \]

(a) [10 points] Express \( v(t) \) in terms of \( x(t) \) if

\[ V(\omega) = \begin{cases} 
-2\omega, & |\omega| < 2 \\
0, & \text{otherwise.}
\end{cases} \]

(b) [10 points] Express \( y(t) \) in terms of \( x(t) \) if

\[ Y(\omega) = \begin{cases} 
-2(\omega - 2), & 0 < \omega < 4 \\
0, & \text{otherwise.}
\end{cases} \]
5. **[20 points total]** A stereo FM signal is created from the left speaker signal $x_L(t)$ and the right speaker signal $x_R(t)$ by setting the FM stereo signal $z(t)$ to be

$$z(t) = [x_L(t) + x_R(t)] + 2 \cos(76000\pi t) [x_L(t) - x_R(t)].$$

(a) **[8 points]** Find an expression for $Z(\omega)$ in terms of $X_L(\omega)$ and $X_R(\omega)$.

(b) **[6 points]** Suppose that $X_L(\omega)$ and $X_R(\omega)$ are as follows, where $\omega_0 = 15000\pi$.

![Diagram of X_L(\omega) and X_R(\omega)]

Sketch $Z(\omega)$ on the axes on the next page.

(c) **[6 points]** The mono FM signal is $y(t) = x_L(t) + x_R(t)$ (this is a single channel signal combining both the left and the right speaker signals). Which of the following expressions is equal to $y(t)$ when $x_L(t)$ and $x_R(t)$ have the Fourier transforms shown in part (b) of this problem?

$$y_1(t) = z(t) \ast [30000 \text{sinc}(30000t)]$$
$$y_2(t) = z(t) [30000 \text{sinc}(30000t)]$$
$$y_3(t) = z(t) p_{60000\pi}(t)$$
$$y_4(t) = z(t) p_{60000\pi}(t)$$
EXTRA WORKSHEET (indicate problem number clearly)
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