Name: ____________________________________________

The examination lasts for 75 minutes and is closed book, closed notes. No electronic devices are permitted, including but not limited to calculators, cellphones, and other handheld devices. (Any such items in the examination room must be off and put away, subject to a 20 point penalty for the first violation and a score of 0 on the exam for the second violation.)

Do all your work on the pages in this exam booklet. **Do not unstaple these pages. Any unstapled or restapled pages will NOT be graded.** You may write on the backs of the pages if you need to. There is an extra worksheet for problem 5 immediately following that problem, and attached at the back of the exam booklet are two extra work pages. (Clearly mark the problem number associated with your work if you use these last two pages.)

Show your work and clearly indicate your final answers. Neatness and organization in your work is important and will influence your grade.

Each problem is weighted toward the final total as shown below.

**Grades**

1. ______________________ (18 pts.)
2. ______________________ (18 pts.)
3. ______________________ (20 pts.)
4. ______________________ (20 pts.)
5. ______________________ (24 pts.)

Total ______________________ (100 pts.)

Two blank pages are provided at the back for extra work space. If you use these, clearly mark which problems are being worked there.

You might find the following formula useful.

\[ \sum_{k=\ell}^{m} r^k = \frac{r^\ell - r^{m+1}}{1 - r} \]
1. [18 points; 3 points each] Mark the following statements as either TRUE or FALSE by circling the appropriate response.

(a) The system with input signal $x$ and output signal $y$ whose input-output relationship is

$$y(t) = \int_{-\infty}^{+\infty} x(\lambda) h(t - \lambda) \, d\lambda$$

is linear, regardless of the nature of $h(t)$.

TRUE \hspace{1cm} FALSE

(b) A discrete-time signal expressed as

$$x[n] = \sum_{k=0}^{\infty} a_k \delta[n - k]$$

may equivalently be expressed as

$$x[n] = a_0 u[n] + (a_1 - a_0) u[n - 1] + (a_2 - a_1) u[n - 2] + (a_3 - a_2) u[n - 3] + \cdots$$

TRUE \hspace{1cm} FALSE

(c) The system

$$y(t) = \int_{-\infty}^{+\infty} e^{-(t-\lambda)} u(t - \lambda) x(\lambda) \, d\lambda$$

is the same as the system

$$y(t) = \int_{-\infty}^{+\infty} e^{-\lambda} u(\lambda) x(t - \lambda) \, d\lambda$$

TRUE \hspace{1cm} FALSE
(d) The system whose input-output relationship is given by
\[ y(t) = t^2 x(t) + (t - 1)^2 x(t - 1) + (t - 2)^2 x(t - 2) \]
is a linear system.

TRUE FALSE

(e) The system whose input-output relationship is given by
\[ y(t) = tx^2(t) + (t - 1)x^2(t - 1) + (t - 2)x^2(t - 2) \]
is a linear system.

TRUE FALSE

(f) For a linear time-invariant (LTI) system, if when the input \( x(t) = x_0(t) = \delta(t) \)
the output is \( y(t) = y_0(t) = v(t) \), then the system’s input-output relationship for
a general input signal \( x(t) \) is
\[ y(t) = v(t) \ast x(t) \]

TRUE FALSE
2. [18 points; 9 points each]

(a) Sketch the discrete-time signal


(b) Sketch the discrete-time signal

3. [20 points] Express $x(t)$ in

$$x(t) = -2 - 1 - 4 - 3 - 1 2 3 4 5 6 x(t)$$

as a sum of steps and ramps.
4. **[20 points]** Let $x[n]$ be the amount you deposit (in thousands of dollars) in year $n$ into a mutual fund retirement account that generates a 6% annual return on investment. If you make an initial deposit (in year 0) of $1,000 but make no additional investments (that is, you set $x[n] = \delta[n]$), the balance $y[n]$ for year $n$ is given by $y[n] = (1.06)^n u[n]$. Note that this investment “system” is linear and time-invariant, and the previous statement implies that its pulse response is $h[n] = (1.06)^n u[n]$. Suppose now that you invest $1,000 each year starting in year 0 (that is, $x[n] = 1$ for $n \geq 0$). Determine an expression for the yearly balance $y[n]$ in this case.
5. [24 points; 12 points each] Find \( y(t) = h(t) \ast x(t) \) when

(a)

\[
\begin{align*}
    h(t) &= e^{-t}u(t) \\
    x(t) &= e^{-2t}u(t)
\end{align*}
\]

(b)

\[
\begin{align*}
    h(t) &= e^{-t}u(t) \\
    x(t) &= e^{-2t} [u(t) - u(t - 2)]
\end{align*}
\]
Extra worksheet for Problem 5.
Extra worksheet.
Extra worksheet.