Instructions:

The examination lasts for 120 minutes. The exam is open book (you can bring your textbook and use it during the exam), and you are allowed to use in the exam two sheets of notes, both sides (8 1/2” x 11” paper). No electronic devices are permitted, including but not limited to calculators, cellphones, and other handheld devices.

Do all your work on the pages in this exam booklet. Do not unstaple these pages. Any unstapled or restapled pages will NOT be graded. There is an extra work page immediately following each problem. If for some reason you need more space than this, write on the backs of the pages (please clearly mark the problem number for such work).

Show your work and clearly indicate your final answers. Neatness and organization in your work is important and will influence your grade.

Each problem is weighted toward the final total as shown below.

Grades

1. ______________________ (20 pts.)
2. ______________________ (20 pts.)
3. ______________________ (20 pts.)
4. ______________________ (20 pts.)
5. ______________________ (20 pts.)
Total ____________________ (100 pts.)

Note that the text book defines a rectangular pulse by the expression

\[ p_r(t) = \begin{cases} 
1, & -\frac{\tau}{2} \leq t \leq \frac{\tau}{2} \\
0, & \text{all other } t 
\end{cases} \]
1. **[20 points]** An LTI discrete-time system has pulse response

\[ h[n] = \begin{cases} 
1, & n = 0 \\
1/2, & n = \pm 1 \\
0, & \text{otherwise} 
\end{cases} \]

(a) What is the system’s frequency response?

(b) If the input signal is \( x[n] = 1 \) for all \( n \), what is the output \( y[n] \)?
Extra worksheet for problem 1
2. [20 points] A signal \(x(t)\) has Fourier transform
\[
X(\omega) = \frac{1}{9 - \omega^2 + j6\omega}.
\]
Find the Fourier transform \(Y(\omega)\) of \(y(t)\) when
\[
y(t) = \frac{dx(t)}{dt} + 3x(t).
\]
3. [20 points] The periodic signal $x(t)$

has a Fourier series given by

$$x(t) = 1 + \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{1}{k} \sin\left(\frac{k\pi}{2}\right) \cos(k\pi t).$$

If $x(t)$ is the input to an LTI system with impulse response

$$h(t) = \frac{3}{2} \text{sinc}\left(\frac{3t}{2}\right),$$

find the Fourier series of the output signal $y(t)$. 
Extra worksheet for problem 3
4. [20 points] You are going to send a communications signal \( x(t) = \cos(\omega_0 t) \) over a channel

\[
\begin{array}{ccc}
\hat{\mathbf{x}}(t) & \hat{\mathbf{H}}(\omega) & \hat{\mathbf{y}}(t) \\
\end{array}
\]

with frequency response

\[
H(\omega) = \frac{(9 - \omega^2)(4 - \omega^2)}{(j\omega + 1)^4}.
\]

You get to choose the input signal’s frequency \( \omega_0 \) as either 1, 3, or 5.

(a) If you want \( y(t) \) to have the largest amplitude, what value of \( \omega_0 \) do you choose?

(b) For your choice of \( \omega_0 \), what is \( y(t) \)?
Extra worksheet for problem 4
5. [20 points] You control a DC motor’s speed by applying a voltage to its windings that is a sum of two terms, one proportional to the difference between a reference speed (input) $x(t)$ and the motor speed (output) $y(t)$ and one proportional to the integral of that difference. Your design of the control results in a differential equation

$$\frac{d^2 y(t)}{dt^2} + 5 \frac{dy(t)}{dt} + 6 y(t) = 4 \frac{dx(t)}{dt} + 6 x(t)$$

that describes the system behavior.

(a) Find the output speed $y(t)$ if $x(t) = u(t)$ and $\dot{y}(0^-) = y(0^-) = x(0^-) = 0$.

(b) Find the output speed $y(t)$ if $x(t) = u(t)$, $\dot{y}(0^-) = 0$, $y(0^-) = 1$, and $x(0^-) = 1$. 
Extra worksheet for problem 5