Name: _____________________________________________

Instructions:
The examination is perhaps somewhat longer than one that would last for 75 minutes, but it shows the kind of problems that will appear on the actual examination. The actual examination will last 75 minutes and will be closed book, closed notes. No calculators permitted. A table of properties of the Fourier transform is attached for your convenience.

Do all your work on the pages in this exam booklet. Do not unstaple these pages. Any unstapled or restapled pages will NOT be graded. You may write on the backs of the pages if you need to, and attached at the back of the exam booklet are two extra work pages.

Show your work and clearly indicate your final answers. Neatness and organization in your work is important and will influence your grade.

Each problem is weighted toward the final total as shown below.

Grades

1. ________________ (10 pts.)
2. ________________ (15 pts.)
3. ________________ (15 pts.)
4. ________________ (15 pts.)
5. ________________ (15 pts.)
6. ________________ (15 pts.)
7. ________________ (15 pts.)
Total ________________ (100 pts.)
Some properties of the Fourier transform

<table>
<thead>
<tr>
<th>Property</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Linearity</strong></td>
<td>$ax(t) + bv(t) \longleftrightarrow aX(\omega) + bX(\omega)$</td>
</tr>
<tr>
<td><strong>Time shift</strong></td>
<td>$x(t - c) \longleftrightarrow X(\omega)e^{-j\omega c}$</td>
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<tr>
<td><strong>Time scaling</strong></td>
<td>$x(at) \longleftrightarrow \frac{1}{a}X\left(\frac{\omega}{a}\right)$, for $a &gt; 0$</td>
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<tr>
<td><strong>Time reversal</strong></td>
<td>$x(-t) \longleftrightarrow X(-\omega)$</td>
</tr>
<tr>
<td><strong>Multiplication by a power of $t$</strong></td>
<td>$t^n x(t) \longleftrightarrow j^n \frac{d^n}{d\omega^n} X(\omega)$, $n = 1, 2, \ldots$</td>
</tr>
<tr>
<td><strong>Multiplication by sinusoids</strong></td>
<td>$e^{j\omega_0 t} x(t) \longleftrightarrow X(\omega - \omega_0)$, for $\omega_0$ real</td>
</tr>
<tr>
<td><strong>Differentiation</strong></td>
<td>$\frac{d^n}{d\omega^n} x(t) \longleftrightarrow (j\omega)^n X(\omega)$, $n = 1, 2, \ldots$</td>
</tr>
<tr>
<td><strong>Integration</strong></td>
<td>$\int_{-\infty}^{t} x(\lambda)d\lambda \longleftrightarrow \frac{1}{j\omega} X(\omega) + \pi X(0)\delta(\omega)$</td>
</tr>
<tr>
<td><strong>Convolution</strong></td>
<td>$x(t) * v(t) \longleftrightarrow X(\omega)V(\omega)$</td>
</tr>
<tr>
<td><strong>Multiplication</strong></td>
<td>$x(t)v(t) \longleftrightarrow \frac{1}{2\pi} X(\omega) * V(\omega)$</td>
</tr>
<tr>
<td><strong>Duality</strong></td>
<td>$X(t) \longleftrightarrow 2\pi x(\omega)$</td>
</tr>
</tbody>
</table>
1. (a) Write down the formula for the Fourier transform $X(\omega)$ of the signal $x(t)$.

(b) Write down the formula for $x(t)$ in terms of its Fourier transform $X(\omega)$. 

2. The trigonometric and complex exponential Fourier series for a periodic signal \( x(t) \) with period \( T \) are

\[
x(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + \sum_{k=1}^{\infty} b_k \sin(k\omega_0 t),
\]

\[
x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}.
\]

(a) Show that

\[
c_k = \begin{cases} 
\frac{1}{2}(a_k - jb_k), & k \geq 1 \\
a_0, & k = 0 \\
\frac{1}{2}(a_k + jb_k), & k \leq 1. 
\end{cases}
\]

(b) If \( x(t) \) is an odd function of time, what can be said about the values of \( \{c_k\} \)?

(c) How is \( \omega_0 \) determined from \( x(t) \)?
3. For each of the signals $x_1(t), x_2(t), x_3(t),$ and $x_4(t)$ shown below, state whether the signal is an even function of time, an odd function of time, or neither even nor odd.
4. The Fourier transform $X(\omega)$ of $x(t)$ is shown below.

Sketch $V(\omega)$ for the following functions defined in terms of $x(t)$.

(a) $v(t) = x(t) \cos(4t)$
(b) $v(t) = x(t) + x(t + 3) + x(t - 3)$
5. If

\[ X(\omega) = 2 \left( \frac{1}{2} + j \frac{\sqrt{3}}{2} \right) \delta(\omega - 2) + 2 \left( \frac{1}{2} - j \frac{\sqrt{3}}{2} \right) \delta(\omega + 2), \]

what is \( x(t) \)?
6. If

\[ x(t) = \begin{cases} 
\cos(\pi t), & -\frac{1}{2} \leq t \leq \frac{1}{2}, \\
0, & \text{otherwise}
\end{cases} \]

what is \( X(\omega) \)?
7. The Fourier transform of
\[ x(t) = u(t - \frac{1}{2}) - u(t + \frac{1}{2}) \]
is
\[ X(\omega) = \frac{\sin(\omega/2)}{\omega/2}. \]
Sketch a plot of \( v(t) \) if
\[ V(\omega) = \frac{\sin^2(\omega/2)}{\omega^2/4}. \]
EXTRA WORKSHEET (indicate problem number clearly)
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