Name: ____________________________________________________________

Instructions:

The examination lasts for 75 minutes and is closed book, closed notes. No calculators permitted. A table of properties of the Fourier transform is attached for your convenience, as is a brief table of Fourier transform pairs, and also a couple of trigonometric identities. There are five problems on the exam.

Do all your work on the pages in this exam booklet. **Do not unstaple these pages. Any unstapled or restapled pages will NOT be graded.** You may write on the backs of the pages if you need to, and attached at the back of the exam booklet are two extra work pages.

Show your work and clearly indicate your final answers. Neatness and organization in your work is important and will influence your grade.

Each problem is weighted toward the final total as shown below.

**Grades**

1. __________________ (20 pts.)
2. __________________ (20 pts.)
3. __________________ (20 pts.)
4. __________________ (20 pts.)
5. __________________ (20 pts.)
Total ______________ (100 pts.)
### Some properties of the Fourier transform

<table>
<thead>
<tr>
<th>Property</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Linearity</strong></td>
<td>$ax(t) + bv(t) \leftrightarrow aX(\omega) + bX(\omega)$</td>
</tr>
<tr>
<td><strong>Time shift</strong></td>
<td>$x(t - c) \leftrightarrow X(\omega)e^{-j\omega c}$</td>
</tr>
<tr>
<td><strong>Time scaling</strong></td>
<td>$x(at) \leftrightarrow \frac{1}{a}X\left(\frac{\omega}{a}\right)$,  for $a &gt; 0$</td>
</tr>
<tr>
<td><strong>Time reversal</strong></td>
<td>$x(-t) \leftrightarrow X(-\omega)$</td>
</tr>
<tr>
<td><strong>Multiplication by a power of $t$</strong></td>
<td>$t^n x(t) \leftrightarrow j^n \frac{d^n}{d\omega^n} X(\omega)$,  $n = 1, 2, \ldots$</td>
</tr>
<tr>
<td><strong>Multiplication by sinusoids</strong></td>
<td>$e^{j\omega_0 t} x(t) \leftrightarrow X(\omega - \omega_0)$,  for $\omega_0$ real</td>
</tr>
<tr>
<td><strong>Differentiation</strong></td>
<td>$\frac{d^n}{d\omega^n} x(t) \leftrightarrow (j\omega)^n X(\omega)$,  $n = 1, 2, \ldots$</td>
</tr>
<tr>
<td><strong>Integration</strong></td>
<td>$\int_{-\infty}^{t} x(\lambda)d\lambda \leftrightarrow \frac{1}{j\omega} X(\omega) + \pi X(0)\delta(\omega)$</td>
</tr>
<tr>
<td><strong>Convolution</strong></td>
<td>$x(t) * v(t) \leftrightarrow X(\omega)V(\omega)$</td>
</tr>
<tr>
<td><strong>Multiplication</strong></td>
<td>$x(t)v(t) \leftrightarrow \frac{1}{2\pi} X(\omega) * V(\omega)$</td>
</tr>
<tr>
<td><strong>Duality</strong></td>
<td>$X(\omega) \leftrightarrow 2\pi x(t)$</td>
</tr>
</tbody>
</table>

### Some Fourier transform pairs

- $\delta(t) \leftrightarrow 1$
- $u(t) \leftrightarrow \frac{1}{j\omega} + \pi \delta(0)$
- $e^{-at}u(t)$, $a > 0 \leftrightarrow \frac{1}{j\omega + a}$

### Some trigonometric identities

- $\sin(\alpha \pm \beta) = \sin(\alpha) \cos(\beta) \pm \cos(\alpha) \sin(\beta)$
- $\cos(\alpha \pm \beta) = \cos(\alpha) \cos(\beta) \mp \sin(\alpha) \sin(\beta)$
- $\sin(\alpha) \cos(\beta) = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$
- $\cos(\alpha) \cos(\beta) = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$
- $\sin(\alpha) \sin(\beta) = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$
1. [20 points] Fill in the four blanks in the following statement.

The formulas

\[ x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} \]  \hspace{1cm} (1)

and

\[ x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} \, dt \] \hspace{1cm} (2)

each express \( x(t) \) as a \underline{Fourier series} of \underline{complex exponentials}. We use (1) if \( x(t) \) is a \underline{periodic} signal. To use (2), we need

\[ \int_{-\infty}^{\infty} |x(t)| \, dt \]

to be \underline{finite}. 

2. [20 points] Find the trigonometric Fourier series for the signal $x(t)$ shown below. Over the interval from $t = 0$ to $t = \pi$, we have $x(t) = \sin(t)$. 

![Graph of the signal x(t)]
3. [20 points] Suppose that

\[ X(\omega) = 3 \left( -\frac{\sqrt{2}}{2} + j \frac{\sqrt{2}}{2} \right) \delta(\omega - 5) + 3 \left( -\frac{\sqrt{2}}{2} - j \frac{\sqrt{2}}{2} \right) \delta(\omega + 5). \]

What is \( x(t) \)?
4. [20 points] Find the signal $x(t)$ corresponding to the Fourier transform

$$X(\omega) = \frac{3}{j\omega + 2} + \frac{3}{j\omega - 2}.$$
5. **[20 points]** You send out a radar signal $x(t)$ and receive a return echo $y(t) = \alpha x(t-\Delta)$ where $\alpha > 0$ is an attenuation factor and $\Delta$ is a delay time. $|X(\omega)|$ and $|Y(\omega)|$ are plotted in the top figure below, and $\angle X(\omega)$ and $\angle Y(\omega)$ are plotted in the bottom figure. What are $\alpha$ and $\Delta$?
EXTRA WORKSHEET (indicate problem number clearly)
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