Capacity of Cooperative Wireless Networks Using Multiple Channels

Devu Manikantan Shila, Yu Cheng and Tricha Anjali
Dept. of Electrical and Computer Engineering,
Illinois Institute of Technology, Chicago, USA

***Abstract*—The existence of channel variations (or fading) is one of the crucial challenges that affects the capacity of wireless networks. Recently, cooperative communications have emerged as a promising approach to achieve spatial diversity and thereby reduce the negative effects of fading on wireless channels. On the other hand, in addition to channel variations, it is well-known that interference among concurrent transmissions also severely limits the network capacity particularly in multi-hop settings. Recent studies indicate that the use of multiple channels can reduce the wireless interference and thus greatly improve the overall network capacity. In this paper, we propose a model termed as CoopMC which employs multiple channels in cooperative networks to mitigate the impact of interference. This work primarily investigates the capacity of CoopMC in multi-hop settings and derive the asymptotic capacity bounds under random placements of nodes. Our analysis reveals the important insights on when a network can benefit from cooperative communications and how multi-channel networking can further improve the network capacity.**

I. Introduction

One of the biggest dilemmas in practical wireless networks is how to provide robust communication over fading channels. Spatial diversity, in the form of employing multiple transmitter-receiver (transceiver) antennas (e.g., multiple input multiple output (MIMO) [1]-[2],[5],[6] is shown to be very effective in coping fading in wireless channel. However, equipping a wireless node with multiple antennas may not be practical, particularly handheld wireless devices, and nodes carrying one or two antennas significantly limit the effectiveness of MIMO techniques [2]-[6]. Recent studies address this limitation through the use of a new paradigm known as *cooperative communications*—also known as virtual MIMO systems—that draws from the idea of using the broadcast nature of the wireless channel to achieve spatial diversity. Under cooperative communications, nodes equipped with a single transceiver (*cooperative relays*) captures a neighboring source's transmission and relay it to the designated destination. The destination combines multiple streams of the same information from both source and relay nodes to recover the original information with high probability. Therefore, by cooperatively relaying the information to the destination, nodes equipped with single antenna achieve the same advantages as those found in MIMO systems.

Due to the advances in cooperative communication, many efforts have been spent in understanding and improving the benefits of deploying cooperative relays in wireless networks, including network capacity analysis, optimal power allocation, resource allocation and relay assignment [3]-[5]. However, most of these works on cooperative relaying have been mainly studied from the standpoint of small-scale wireless networks—typically a network with a single source, destination and relay—which may not be realistic, especially for multi-hop wireless networks. A few works in [4]-[6] address this limitation and demonstrate that deploying cooperative relays in large-scale wireless network in fact incurs several challenges. For instance, Zhu *et. al* [4] shows both analytically and experimentally that deploying cooperative relays in large-scale wireless networks can lead to an elevated level of interference which in turn results in degraded throughput and higher packet losses. On the other hand, in the last few years studies [7], [8] show that employing multiple channels in wireless networks can mitigate the negative effects of interference and thus substantially enhance the performance of wireless networks.

Motivated by these ideas, it is thus worth investigating the benefits of integrating multiple channels into large-scale cooperative wireless networks. In this paper, we address this problem and propose a model known as *cooperative network with multiple channels* (CoopMC) which deploys cooperative relays in large-scale networks and uses multiple channels to reduce the impact of interference in such networks. Specifically, we focus on deriving the capacity of CoopMC model and reveal the important insights on when a network can benefit from cooperative communications and how multi-channel networking can further improve the network capacity. To the best of our knowledge, this is the first effort to quantify the capacity of cooperative networks operating on multiple channels.

The remainder of the paper is as follows. We begin with the description of our model, CoopMC, in section II and then discuss the performance of cooperative transmissions in terms of both the throughput and interference region. In section III, we provide the analytical results of the throughput capacity of random networks and compare the capacity under cases of multi-channel cooperative networks, single-channel cooperative networks, and traditional non-cooperative networks. We conclude our work in section IV.

II. CoopMC Model

We consider a static wireless network composed of n nodes, with each node being either a source node (s) or a destination node (d). Contrary to direct transmission (*i.e.*, no cooperation), in cooperative transmission relays (r) are assigned to assist the communication between the source and destination. Hence, we assume that besides n nodes there are sufficient number of relays in the network, $\Theta(n^2)$, and each $s-d$ link is assigned with a single relay to aid the communication process. We further consider the following assumptions for our model.

(A.i) There are C channels in the network and each node (source, destination and relay) is equipped with a single half-duplex interface. We use the $(1, C)$ notation to denote these

networks.
(A.ii) Due to half-duplex nature, an interface can either transmit or receive data on any one channel at a given time. A cooperative link consists of a direct path $s-d$, and a relay path $s-r-d$. This paper considers that direct path and relay path use the same channel. Thus, a transmission from source to destination via a relay path is completed in two time slots.
(A.iii) Each channel can support a maximum data rate of W_{coop} (W_{direct}) bits/sec under cooperative (direct) communication, independent of the number of channels. Therefore, the aggregate data rate possible by using all C channels under both cooperative and direct communication are CW_{coop} and CW_{direct} respectively.

Moreover, since the main aim of this work is to understand the benefits of CoopMC model, we do not deal with issues pertaining to resource/power allocation or relay selection. Please see [6] for further details regarding these issues.

A. Throughput of Cooperative and Direct Transmissions

In [3], Laneman *et.al* characterizes the performance of several cooperative relaying protocols such as AF (Amplify-and-Forward), DF (Decode-and-Forward) in terms of outage and ergodic capacities; the study primarily manifests the benefits of cooperative transmission—in the absence of interference—as opposed to direct transmission in the presence of fading. Due to space limitations, we focus on the outage capacity of AF relay[1] and use throughput, which is the product of outage capacity and spectrum bandwidth (B) Hz, to determine the quality of transmissions between source and destination in the presence of relay. For baseline comparison, we also provide the results for direct communication model. Hence, the throughput of both cooperative (W_{coop}) and direct communication (W_{direct}) model are expressed as follows:

$$W_{coop} = \frac{B}{2}.\Psi_{coop},\; W_{direct} = B.\Psi_{direct} \qquad (1)$$

where the factor $1/2$ in W_{coop} is based on the assumption (A.ii). Ψ_{coop} and Ψ_{direct} respectively are the outage capacities of cooperative and direct transmissions. Loosely speaking, outage capacity measures the robustness of the transmissions to fading. Thus, given the probability of outage, ρ, the outage capacities are expressed as follows [3]:

$$\Psi_{coop} = \log_2(1+\gamma\sqrt{\rho\phi}),\; \Psi_{direct} = \log_2(1+\gamma\rho\sigma_{sd}^2) \qquad (2)$$

where γ represents the transmit signal-to-noise ratio and $\phi = 2\sigma_{sd}^2(\frac{\sigma_{sr}^2.\sigma_{rd}^2}{\sigma_{sr}^2+\sigma_{rd}^2})$. The parameter σ_{ij}^2 is the fading variance between nodes i and j. This work focuses on the special case of symmetric networks in which the fading variances are identical. Specifically, we assume that each cooperative link can transfer data at the rate of W_{coop} (W_{direct} for direct link) on channel x, where $x \in C$, provided that there are no interfering links transmitting on channel x at the same time.

[1] In AF mode, the relay amplifies the information received from the source and then forwards it to the destination without demodulation or decoding. It is to be noted that the results in this work are applicable for other relaying protocols and ergodic capacities as well.

B. Link-based Interference Model

In sec. II-A, we quantified the throughput of cooperative link under the assumption of "absence of interference". However, this assumption is not valid in practice, especially when relays are deployed in multihop networks. We therefore study the impact of interference of such networks by using the protocol model proposed in [9]. In this model, a node v_j will not receive the data from a transmitter v_i correctly if node v_j lies within the interference region of another actively transmitting node v_k, *i.e.*, $\|v_j - v_k\| \leq X_I$. This work mainly focuses on the link-based protocol interference model[2] rather than receiver or transmitter based interference models studied in [4]. Further, we assume an uniform interference and communication range, X_I and X_S respectively, for all nodes—based on the assumption of a fixed transmit power for all nodes, and add the following two assumptions to our interference model: (A.iv) For the simplicity of analysis, we model the interference region of a link by choosing the center of the link as the disk center and then approximating the interfering region with a large disk, see fig 1; (A.v) Since the area of the interference region depends on the distance between the communicating pairs, we focus on the scenario where each communicating pair is placed at a maximum distance apart. Consequently, in CoopMC the relay node is placed at a distance of $X_S d_r$ from both source and destination, *i.e.*, $\|s-r\| = \|r-d\| = X_S d_r$.

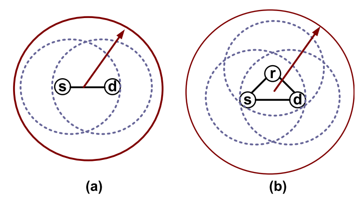

Fig. 1. The interference region of (a) $s-d$ and (b) $s-r-d$ links.

Now, we approximate the interference region of our baseline model *i.e.*, direct link $s-d$ with a disk of radius $\delta_d.X_S$. $\delta_d.X_S$ is given by $X_S + X_I = (1+\Delta)X_S$, where $\Delta = X_I/X_S$ is the interference ratio. The parameter $X_I + X_S$ ensures that any two links transmitting in some channel c' will not conflict with each other if their median are $X_I + X_S$ apart. Conversely, we say that two links are disjoint if the centers of the both disks are farther away by $\delta_d.X_S$. Hence, we represent each $s-d$ link with a disk of radius $\delta_d.X_S/2$—property of Unit Disk Graph [4].

Similarly, we approximate the interference region of cooperative transmission with a disk of radius $\delta_c.X_S$; $\delta_c.X_S$ is determined as:

$$\delta_c.X_S = \begin{cases} (1+\Delta)X_S & : 1/2 < d_r < 1/\sqrt{2}; \\ (\Delta + \frac{d_r^2}{2\sqrt{d_r^2 - 1/4}} + \frac{1}{2})X_S & : 1/\sqrt{2} \leq d_r \leq \Delta; \end{cases}$$

where, each cooperative link is associated with a disk of radius $\delta_c X_S/2$ centered at the median of s, d and r (see fig. 1).
From the value of $\delta_c.X_S$, it is clear that when the cooperative relay is placed close to the link $s-d$, the perceived amount of interference is same as that of direct transmission, i.e., $\delta_c.X_S =$

[2] In link based interference model, we consider the interference region around a link. Conversely, when a link is active, no nodes in the vicinity of either sender or receiver of the link can take part in other communication process. Thus, we define the interference region of a link as the union of the interference region of each communicating pair.

$\delta_d.X_S$; however, when the relay is far apart from the link $s-d$, i.e., $d_r \geq 1/\sqrt{2}$, it incurs an increased level of interference. Since the intensity of interference depends on the relay distance d_r, we represent the radius of interfering region under CoopMC model as $\delta.X_S = \max(\delta_c.X_S, \delta_d.X_S)$.

Note A: In the sequel, we assume each cooperative link as a $s-d$ link with an interference disk of radius $\delta.X_S/2$ and a maximum data rate of W_{coop}. This assumption simplifies our analysis so that we can focus on understanding the benefits of CoopMC model.

III. Throughput Capacity for Random Networks

Under the assumption in Note A (Section II-B), we consider that n nodes are randomly located on the surface of a torus of unit area. Each node selects a destination randomly to which it transmits $\lambda_{coop}(n)$ bits/sec. The maximum value of $\lambda_{coop}(n)$ that can be supported by every source-destination pair with high probability (*whp*)[3] is defined as the per-node throughput of the network [7], [9]. Since there are total of n flows[4], the network capacity is defined to be $n\lambda_{coop}(n)$.

A. Upper Bound

The capacity of CoopMC model under random network setting is limited by the following three constraints [7].

1) *Connectivity constraint:* In random networks, this constraint is necessary to ensure that the network is connected *whp*. Previous work [9] shows that $X_S(n) > \sqrt{\frac{\log n}{\pi n}}$ is necessary to guarantee connectivity *whp*. From the interference model, we know that the number of concurrent transmissions possible on any single channel is limited to $\frac{1}{(\pi(\delta X_S/2)^2)} = \frac{4}{\pi\ (\delta X_S)^2}$. In addition, since each source-destination of a flow is separated by an average of $\Theta(1)$ (we assume a torus of area $1m^2$) distance, we have the average number of hops as $\Theta(\frac{1}{X_S(n)})$ between each source-destination pair (see [9] for details). Thus, the network capacity using all C channels is limited to $O(\frac{CW_{coop}}{\delta^2 X_S(n)})$. Substituting $X_S(n) > \sqrt{\frac{\log n}{\pi n}}$, we obtain the upper bound for network capacity as $O(\frac{CW_{coop}}{\delta^2}\sqrt{\frac{n}{\log n}})$.

2) *Interference constraint:* The capacity of random cooperative networks using multiple channels is also constrained by interference.

Lemma 1. *The network capacity of random networks is atmost $O(\frac{W_{coop}}{\delta}\sqrt{nC})$ bits/sec for CoopMC model.*

Proof: We adopt the reasoning introduced in [9] to get the upper bound for the transport capacity of the network. We assume that each source node originates λ bits/sec. Let the average distance between source and destination pairs be $\bar{L}$ then the transport capacity of the network is given as $\lambda n\bar{L}$ bit-meters/sec. In a time period of length one second, consider a bit b, $1 \leq b \leq \lambda n$. Let us assume that it moves from its origin to its destination in a sequence of $h(b)$ hops, where the hth hop traverses a distance of r_b^h. Then we obtain

$$\sum_{b=1}^{\lambda n}\sum_{h=1}^{h(b)} r_b^h \geq \lambda n\bar{L} \tag{3}$$

[3]In this paper, *whp* implies probability with $\geq 1-1/n$

[4]The traffic from a source node to destination node is called a flow

Let us define H to be the total number of hops traversed by all bits in one second, i.e., $H = \sum_{b=1}^{\lambda n} H(b)$. Therefore, the total number of bits transmitted by all nodes in a second is equal to H. Since each node can transmit over a channel with rate W_{coop}, the total number of bits transmitted by all nodes is atmost $\frac{W_{coop}n}{2}$, where the factor $1/2$ is due to the fact that there are n nodes in the network and thus, we have $n/2$ number of transmitter-receiver pairs in that time period. This yields $H \leq \frac{W_{coop}n}{2}$.

From the interference model introduced above, we see that disks of radius $\delta/2$ times the lengths of hops centered around the links over the same channel in the same slot are essentially disjoint. Since the area consumed on each channel is bounded above by the area of the domain (i.e., torus of area = $1m^2$), summing over all channels, we have the constraint $\sum_{b=1}^{\lambda n}\sum_{h=1}^{h(b)} \frac{\pi\delta^2}{4}(r_b^h)^2 \leq CW_{coop}$, which can be rewritten as follows:

$$\sum_{b=1}^{\lambda n}\sum_{h=1}^{h(b)} \frac{1}{H}(r_b^h)^2 \leq \frac{4CW_{coop}}{\pi\delta^2 H} \tag{4}$$

Note now that the quadratic function is convex. Hence,

$$(\sum_{b=1}^{\lambda n}\sum_{h=1}^{h(b)} \frac{1}{H} r_b^h)^2 \leq \sum_{b=1}^{\lambda n}\sum_{h=1}^{h(b)} \frac{1}{H}(r_b^h)^2 \tag{5}$$

Combining (4) and (5) yields

$$\sum_{b=1}^{\lambda n}\sum_{h=1}^{h(b)} r_b^h \leq \sqrt{\frac{4CW_{coop}H}{\pi\delta^2}} \tag{6}$$

Now substituting $H \leq \frac{W_{coop}n}{2}$ in (6) and using (3) gives $\lambda n\bar{L} \leq \frac{W_{coop}}{\delta}\sqrt{\frac{2nC}{\pi}}$. This proves that the upper bound for the transport capacity of the network is $O(\frac{W_{coop}}{\delta}\sqrt{nC})$ bits-meters/sec. Since in a random network, each of the $(s-d)$ pairs are separated by an average distance of $\Theta(1)$ meter (the area of torus=$1m^2$), we have the network capacity of CoopMC model under random network setting as at most $O(\frac{W_{coop}}{\delta}\sqrt{nC})$ bits/sec. ■

3) *Destination bottleneck constraint:* The network capacity of cooperative network is also restricted by the flows toward a destination node, d.

Lemma 2. *The capacity of random networks under bottleneck constraint is atmost $O(\frac{W_{coop}n}{F(n)})$ bits/sec for CoopMC model where $F(n) = \Theta(\frac{\log n}{\log\log n})$ is the maximum number of flows from other nodes to a chosen destination.*

Proof: Due to space constraints, we omit the proof as it has already been presented in [7]. ■

Combining the three bounds under the three constraints, we obtain that the network capacity is atmost $O(\min(\frac{CW_{coop}}{\delta^2}\sqrt{\frac{n}{\log n}}, \frac{W_{coop}}{\delta}\sqrt{nC}, \frac{W_{coop}n\log\log n}{\log n}))$ and $O(\min(\frac{CW_{direct}}{\delta_d^2}\sqrt{\frac{n}{\log n}}, \frac{W_{direct}}{\delta_d}\sqrt{nC}, \frac{W_{direct}n\log\log n}{\log n}))$bps for CoopMC and baseline model respectively. The minimum of three bounds is used to obtain an upper bound on the capacity.

Theorem 1. *The upper bound on the capacity of a $(1,C)$ random network is as follows.*

1) *When $C = O(\delta^2\log n)$, the network capacity is $O(\frac{CW_{coop}}{\delta^2}\sqrt{\frac{n}{\log n}})$ bits/sec.*

2) *When* $C = \Omega(\delta^2 \log n)$ *and also* $= O(\delta^2 n (\frac{\log\log n}{\log n})^2)$, *the network capacity is* $O(\frac{W_{coop}}{\delta}\sqrt{nC})$ *bits/sec.*
3) *When* $C = \Omega(\delta^2 n (\frac{\log\log n}{\log n})^2)$, *the network capacity is* $O(\frac{W_{coop} n \log\log n}{\log n})$ *bits/sec.* □

B. Constructive Lower bound

To prove that the upper bound in Section III-A can be quite tight, we construct a network and then design a routing scheme and a transmission schedule as follows.

(1) **Torus Division**: We divide the unit-area torus into equal-sized squares (or cells), each of area $s(n)$ where we set $s(n) = \min(\max(\frac{100\log n}{n}, \frac{C}{\delta^2 n}), \frac{1}{F(n)^2})$, $F(n)$ is given by $\Theta(\frac{\log n}{\log\log n})$; see III-A. Specifically, the size of each square must satisfy the three constraints presented in Section III-A: cell size needed to ensure connectivity, interference and destination bottleneck constraint respectively. Note that each cell must contain certain number of nodes to guarantee successful transmission of flow(s) from source node(s) to its (their) intended destination node(s) which is lying in the same cell as that of source node or another cell; we can see that the number of nodes present in a cell in fact depends on the size of each cell. We next bound the number of nodes that are present in each cell of size $s(n)$.

Lemma 3. *If* $s(n)$ *is greater than* $\frac{50\log n}{n}$, *each cell has* $\Theta(ns(n))$ *nodes per cell,* whp.

Proof: This lemma can be proved using well-known results (see Chapter 3, [10]). Due to space constraints, we do not repeat the proof here. ■

To simplify the analysis, we take $s(n) = \frac{100\log n}{n}$ for a large n and thus, Lemma 3 holds *whp*.

Before stating Lemma 4, we make the following definition: We say that cell B interferes with another cell A if a transmission in cell B can affect the success of a simultaneous transmission in cell A. We set the maximum distance over which a node can communicate, X_S, to be $\sqrt{8s(n)}$. Note that with this transmission distance, a node in one cell can communicate with any node in its eight neighboring cells.

Lemma 4. *The number of cells that interfere with any given cell is bounded by a constant* c_1, *i.e, independent of* n *and* $s(n)$.

Proof: Under the link-based interference model, two links are "non-interfering" if the median of two links are separated by $d = \delta.X_S$. Using simple geometric arguments we get the number of interfering cells, c_1, as at most $c_1 \leq 2\frac{d^2}{s(n)} = 16\delta^2$, which is independent of n and $s(n)$. ■

(2) **Routing Scheme**: We construct a simple routing scheme that chooses a route with the shortest distance to forward packets. A straight line, $s-d$, is passing through the cells where nodes s and d are located [here, s refers to the source of the flow and d refers to the final destination of the flow]. Packets are delivered along the cells lying on the $s-d$ line. Then, we choose a node within each cell lying on the straight line to carry that flow. The node assignment is based on load balancing. The flow assignment process is presented below:

(2a) *Assign source and destination nodes*: For any flow that originates from (terminates in) a cell, node s (d) is assigned to the flow. After this step, we are left with the flows passing through a cell. (2b) *Assign remaining flows*: For load balancing, we assign each remaining flow to a node that has been assigned the least number of flows. Thus, each node has nearly the same number of flows.

We use the result in [7] to bound the number of $s-d$ lines passing through any cell. We state their lemma here.

Lemma 5. *The maximum number of lines passing through any cell is* $O(n\sqrt{s(n)})$ whp. □

Based on Lemma 3, we know that each cell has $\Theta(ns(n))$ nodes with *whp*. Besides, each cell has $O(n\sqrt{s(n)})$ flows based on Lemma 5 and hence each node in the network is assigned at most $O(\frac{1}{\sqrt{s(n)}})$ flows [see step 2(b) of routing scheme]. Combining with step 2(a) and destination bottleneck constraint, the total flows assigned to every node is $O(1+F(n)+1/\sqrt{s(n)}$ which is also dominated by $O(1/\sqrt{s(n)})$ (note that $s(n)$ is at most $(1/F(n))^2$, thus $F(n)$ is at most $1/\sqrt{s(n)}$).

(3) **Transmission Scheduling**: We consider a scheduling scheme for a $(1, C)$ network. Any transmissions in this model must satisfy the following constraints: (a) each interface only allows one transmission/reception at the same time; and (b) any two transmissions on any channel should not interfere with each other.

We propose a *time-division multi-access* (TDMA) scheme to schedule transmissions [7], which satisfy the aforementioned constraints. In this scheme, a second is divided into a number of edge-color slots and at most one transmission/reception is scheduled at every node during edge-color slot which satisfies the constraint (a). Further, each edge-color slot is divided into mini-slots and in each mini-slot, each transmission satisfies the aforementioned constraints (a) and (b).

We now describe the two time slots as follows:

(1) *Edge-color slot*: First, we map each cooperative link to direct link (this satisfies assumption in Note A, see Section. II-B) and then construct a routing graph in which vertices represent the nodes in the network and an edge denotes transmission/reception of a node. In this construction, one hop along a flow is associated with one edge in the routing graph. In [11], it is shown that this routing graph can be edge-colored with at most $O(1/\sqrt{s(n)})$ colors. We now divide *one second* into $O(1/\sqrt{s(n)})$ edge-color slots and thus, each edge-color slot has a length of $\Omega(\frac{1}{1/\sqrt{s(n)}})$= $\Omega(\sqrt{s(n)})$ seconds. Since each slot is represented with a unique edge-color, all edges connecting to a vertex use different colors and thus, each node has at most one transmission/reception scheduled in any edge-color time slot.

(2) *Mini-slot*: Second, we divide each edge-color slot into mini-slots. We build a schedule that assigns a transmission to a node in a mini-slot within a edge-color slot over a channel. We construct an interference graph in which nodes represent the vertices of the graph and edges denotes interference between two nodes. Based on Lemma 4, every cell has at most c_1 interfering cells and each cell has $\Theta(ns(n))$ nodes based on Lemma 3. Hence, each node has at most $O(c_1 ns(n))$ edges in the interference graph. It is shown that a graph of degree atmost k can be vertex-colored with at most $k+1$ colors. Therefore, the interference graph can be vertex-colored with at most $O(c_1 ns(n))$ colors. We use $k_1 c_1 ns(n) (= c_2 ns(n))$ to denote the number of vertex-colors (where k_1 is a constant). We

know that two nodes with same vertex color do not interfere with each other while nodes with different colors interfere with each other. Hence, we schedule the interfering nodes either on different channels or on different minislots on the same channel. We divide each edge-color slot into $\left\lceil \frac{c_2 ns(n)}{C} \right\rceil$ mini-slots on every channel and assign the mini-slots on each channel from 1 to $\left\lceil \frac{c_2 ns(n)}{C} \right\rceil$. A node assigned with a color x, $1 \leq x \leq c_2 ns(n)$ is allowed to transmit in mini-slot $\lceil \frac{x}{C} \rceil$ on channel $(x \ mod \ C) + 1$.

Now, we analyze the capacity of the $(1, C)$ network. Recall that each edge-color slot has a length of $\Omega(\sqrt{s(n)})$ seconds and each edge-color slot is further divided into $\left\lceil \frac{c_2 ns(n)}{C} \right\rceil$ mini-slots over every channel. Therefore, each mini-slot has a length of $\Omega(\frac{\sqrt{s(n)}}{\lceil \frac{c_2 ns(n)}{C} \rceil})$. Since each channel can transmit at the rate of W_{coop} bps, in each mini-slot, $\lambda_{coop}(n) = \Omega(\frac{W_{coop}\sqrt{s(n)}}{\lceil \frac{c_2 ns(n)}{C} \rceil})$ can be transported. Since $\left\lceil \frac{c_2 ns(n)}{C} \right\rceil \leq \frac{c_2 ns(n)}{C} + 1$, we have $\lambda_{coop}(n) = \Omega(\frac{CW_{coop}\sqrt{s(n)}}{c_2 ns(n) + C})$ bps. Hence, $\lambda_{coop}(n) = \Omega(\min(\frac{CW_{coop}}{c_2 n\sqrt{s(n)}}, W_{coop}\sqrt{s(n)}))$. Thus, the network capacity $n\lambda_{coop}(n)$ is given by $\lambda_{coop}(n) = \Omega(\min(\frac{CW_{coop}}{c_2\sqrt{s(n)}}, W_{coop} n\sqrt{s(n)}))$ $= \Omega(\min(\frac{CW_{coop}}{\delta^2\sqrt{s(n)}}, W_{coop} n\sqrt{s(n)}))$ bits/sec.

Substituting the size of cell, $s(n) = \min(\max(\frac{100\log n}{n}, \frac{C}{\delta^2 n}), \frac{1}{F(n)^2})$, we have the following theorem.

Theorem 2. *The lower bound on the capacity of a* $(1, C)$ *random network is as follows.*

1) *When* $C = O(\delta^2 \log n)$, $s(n) = \Theta(\frac{\log n}{n})$, *the network capacity is* $\Omega(\frac{CW_{coop}}{\delta^2}\sqrt{\frac{n}{\log n}})$ *bits/sec.*
2) *When* $C = \Omega(\delta^2 \log n)$ *and also* $= O(\delta^2 n(\frac{\log\log n}{\log n})^2)$ *and* $s(n) = \Theta(\frac{C}{\delta^2 n})$, *the network capacity is* $\Omega(\frac{W_{coop}}{\delta}\sqrt{nC})$ *bits/sec.*
3) *When* $C = \Omega(\delta^2 n(\frac{\log\log n}{\log n})^2)$ *and* $s(n) = \Theta(\frac{1}{F(n)^2})$, *the network capacity is* $\Omega(\frac{W_{coop} n \log\log n}{\log n})$ *bits/sec.* □

The lower bound matches with the upper bound (Theorem 1) implying that the bounds are tight and thus, the network capacity is $n\lambda_{coop} = \Theta(\min(\frac{CW_{coop}}{\delta^2}\sqrt{\frac{n}{\log n}}, \frac{W_{coop}}{\delta}\sqrt{nC}, \frac{W_{coop} n\log\log n}{\log n}))$ for CoopMC model. Substituting W_{direct} and δ_d in the above capacity equations, we get the network capacity for direct model as $n\lambda_{direct} = \Theta(\min(\frac{CW_{direct}}{\delta_d^2}\sqrt{\frac{n}{\log n}}, \frac{W_{direct}}{\delta_d}\sqrt{nC}, \frac{W_{direct} n\log\log n}{\log n}))$.

C. Comparison to Non-Cooperative Wireless Networks

We now compare the performance of CoopMC model with the baseline model for the cases where $W_{coop} >$ or $\leq W_{direct}$ and $\delta = \delta_d$ or δ_c. Specifically, we analyze the performance of CoopMC model over baseline model in terms of $\frac{\text{Outage Throughput}}{\text{Interfering Region}}$. We already know that when $W_{coop} \leq W_{direct}$, there is no benefit in employing cooperative relays and thus, we focus on the cases where W_{coop} is greater than W_{direct} and $\delta = \max(\delta_d, \delta_c)$. Figures 2 and 3 depicts the network capacity of cooperative and direct model for the following two cases: $\frac{W_{coop}}{\delta_d^2} > \frac{W_{direct}}{\delta_d^2}$ and $\frac{W_{coop}}{\delta_c^2} < \frac{W_{direct}}{\delta_d^2}$. From these figures, we see that the network capacity of both cooperative and direct model has three regions (or constraints) (A, B, C) that follows from the Theorems 1 and 2. We also note that the capacity of both the models remains constant in region C. This is because in region C all transmissions can be regarded as interference-free and hence, addition of more channels do not add any further benefits. For $\delta_c \geq 1$ and $\delta_d \geq 1$, we now study the following cases:

Case A. $\boldsymbol{\delta = \delta_d}$ and $\boldsymbol{\frac{W_{coop}}{\delta_d^2} > \frac{W_{direct}}{\delta_d^2}}$: In this case, we make two observations: (a) The cooperative transmission $(s - r - d)$ incurs same amount of interference $(\delta = \delta_d)$ as that of direct transmission $(s - d)$ and (b) Cooperative transmission $s - r - d$ has higher benefits over direct transmission in terms of the outage throughput $(W_{coop} > W_{direct})$. From fig. 2, we see that when $1 \leq C < \delta_d^2 n(\frac{\log\log n}{\log n})^2$, the network capacities, $n\lambda_{coop}$ and $n\lambda_{direct}$, indeed increases with the increase in number of channels. However, as mentioned earlier we see that when $C \geq \delta_d^2 n(\frac{\log\log n}{\log n})^2$, the network capacities of both models remains constant. Finally, due to the fact that presence of relay does not incur any additional interference and $W_{coop} > W_{direct}$, we can conclude that cooperative scheme outperforms direct model for $C \geq 1$.

Case B. $\boldsymbol{\delta = \delta_c}$ and $\boldsymbol{\frac{W_{coop}}{\delta_c^2} > \frac{W_{direct}}{\delta_d^2}}$: In this case, though the presence of relay incurs an increased level of interference (δ_c) as opposed to direct transmission (δ_d), we see that CoopMC model in fact achieves a capacity gain of $\frac{W_{coop}\delta_d^2}{W_{direct}\delta_c^2} > 1$ over direct model for $C \geq 1$. This gain is due to the fact that W_{coop} is much higher than W_{direct} and in particular W_{coop} is $> \frac{W_{direct}\delta_c^2}{\delta_d^2}$ and thus, overshadows the negative effects of interference.

Case C. $\boldsymbol{\delta = \delta_c}$ and $\boldsymbol{\frac{W_{coop}}{\delta_c} < \frac{W_{direct}}{\delta_d}}$: In this case, even though $W_{coop} > W_{direct}$, the position of relay indeed leads to an increased level of interference and thus, reduces the capacity of cooperative model by a factor of $\frac{W_{coop}\delta_d^2}{W_{direct}\delta_c^2} < 1$ [Regions A and B, fig. 3]. In fig. 3, we see that when $C < \frac{\delta_c^2 n(\frac{\log\log n}{\log n})^2}{(W_{coop}/W_{direct})^2}$, direct model outperforms cooperative model, that is $n\lambda_{direct} > n\lambda_{coop}$. On the other hand, we also see that when $C > \frac{\delta_c^2 n(\frac{\log\log n}{\log n})^2}{(W_{coop}/W_{direct})^2}$, cooperative model outperforms direct model and further reaches constant at $C \geq \delta_c^2 n(\frac{\log\log n}{\log n})^2$. Intuitively, this improvement is due to the fact that when links operate in multiple channels, interference is no longer a limiting factor [region C in fig. 3] and hence, CoopMC model can outperform direct model when $W_{coop} > W_{direct}$.

Clearly, we see that employing multiple channels $(C > 1)$ can significantly improve the performance of cooperative networks as opposed to single channel case $(C = 1)$ by mitigating the negative effects of interference and thereby increasing the spatial reuse. Importantly, we see that when $W_{coop} > W_{direct}$ and sufficient number of channels are present in the network, we can greatly enhance the performance of cooperative wireless networks.

IV. Conclusion

In contrast to existing studies that either focus on single channel or small-scale cooperative networks, we take a different

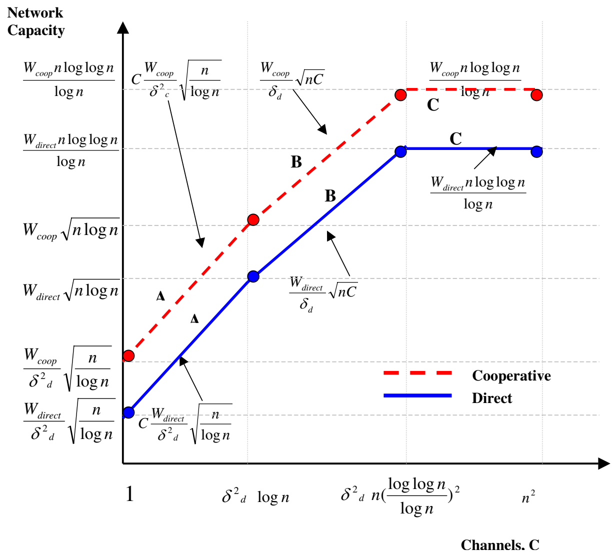

Fig. 2. $\frac{W_{coop}}{\delta_d^2} > \frac{W_{direct}}{\delta_d^2}$ (Figure not to scale)

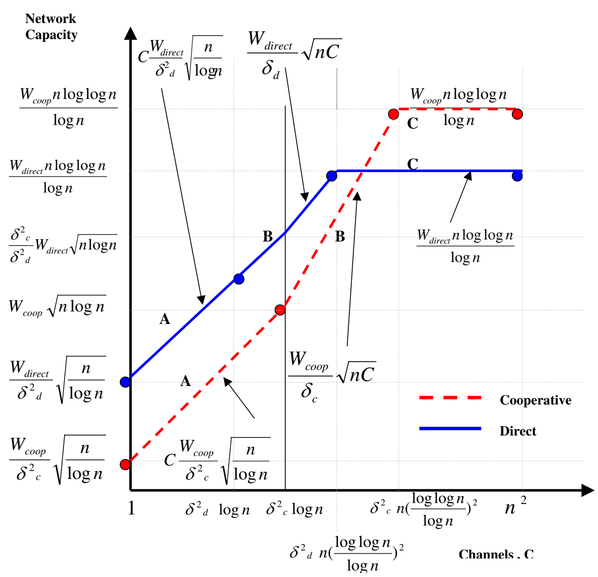

Fig. 3. $\frac{W_{coop}}{\delta_c^2} < \frac{W_{direct}}{\delta_d^2}$ and $W_{coop} > W_{direct}$. Two curves meet at point when $C = \frac{\delta_c^2 n(\frac{\log \log n}{\log n})^2}{(W_{coop}/W_{direct})^2}$ (Figure not to scale)

approach and study a model known as cooperative network with multiple channels (CoopMC) under multi-hop settings. Specifically, we derive the lower and upper bounds on the capacity of CoopMC model under random placements of nodes and study the benefits of this model over baseline model. We show that employing multiple channels can significantly mitigate the impact of interference and thus greatly enhance the performance of cooperative networks.

References

[1] A.J Paulraj, D.A Gore, R.U Nabar, and H. Bolcskei, "An overview of MIMO communications-a key to gigabit wireless," in *IEEE Proceedings*, vol. 92, no. 2, pp. 198- 218, 2004.

[2] A. Kumar, D. Manjunath and J. Kuri. "Wireless Networking," in *Morgan Kaufmann*, pp. 1-448, 2008.

[3] J. N. Laneman, D. N. C. Tse and G.W. Wornell, "Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior," in *IEEE Transactions on Information Theory*, vol. 50, no.12, pp. 3062-3080, 2004.

[4] Y. Zhu and H. Zheng, "Understanding the Impact of Interference on Collaborative Relay," in *IEEE Transactions on Mobile Computing*, vol. 7, no. 6, pp. 724-736, june 2008.

[5] A. Scaglione, D. Goeckel and J. N. Laneman, "Cooperative Communications in Mobile Ad-Hoc Networks: Rethinking the Link Abstraction," in *IEEE Signal Processing Magazine*, vol. 23, no. 5, pp. 18-29, Sept. 2006.

[6] Y. Zhang, H. Chen and M. Guizani, "Cooperative Wireless Communication," in 1^{st} *ed., Auerbach Publications*, pp.1-528, 2009.

[7] P. Kyasanur and N.H. Vaidya, "Capacity of multi-channel wireless networks: impact of number of channels and interface," in *Proc. MOBICOM*, pp. 43-57, August 28-September 02, 2005, Cologne, Germany.

[8] A. Raniwala and T. Chiueh, "Architecture and algorithms for an IEEE 802.11-based multi-channel wireless mesh network," in *Proc. of IEEE INFOCOM*, vol.3, pp. 2223- 2234, 13-17 March 2005.

[9] P. Gupta and P.R. Kumar, "The Capacity of Wireless Networks," in *IEEE Transactions on Information Theory*, 46(2), pp. 388-404, 2000.

[10] R. Motwani and P. Raghavan, "Randomized algorithms," in *Cambridge Univ. Press*, 1995.

[11] D.B.West, "Introduction to graph theory," in 2^{nd} *ed., N.J.: Prentice Hall PTR*, 2001.