On the Performance of a Hybrid Frequency and Phase Shift Keying Modulation Technique

Ramon A. Khalona, Guillermo E. Atkin, and Joseph L. LoCicero

***Abstract*— In this paper, we present the performance of a hybrid modulation technique derived from binary frequency shift keying. In the hybrid scheme, each frequency is allowed to be phase modulated with any of p discrete, equally spaced phase shifts. The spectral separation between frequencies, Δf, is chosen to make the carriers orthogonal, thus generating a multidimensional modulation scheme. The performance of complete and expurgated phase codes is examined for the Gaussian and Rayleigh fading channels, and their spectral characteristics are determined.**

I. Introduction

THE performance of binary frequency shift keying has been examined [1] as a function of the deviation ratio, $h = (f_2 - f_1)T$, where f_1 and f_2 are the binary frequencies and T the signaling interval. Specifically, it has been shown that the choice $h = 0.5$ (known as minimum shift keying) yields a modulation scheme with good power and spectral efficiency that can be demodulated coherently with relative simplicity.

We introduce a combined frequency and phase shift keying (FPSK) modulation technique by considering the *simultaneous* transmission of several phase-modulated frequencies. The power-bandwidth performance is derived as a function of the number of phase shifts, p, allowed for each frequency when $h = 1$. This value of the deviation ratio ensures orthogonality between the frequencies, thereby allowing individual detection of each frequency without cross-interference. In general, for N transmitted phase-modulated carriers, the orthogonality between frequencies gives rise to a $2N$-dimensional signal space. Our attention is focused on the case $N = 2$, and a four-dimensional (4-D) signal space. The signal design problem for this dimensionality has been treated previously [2], [3] for signals with unequal energies under a peak or average energy constraint. The analysis in this paper is limited to equal energy signals where the receiver is very easily implemented.

Section II examines the signal structure, its correlation properties, and performance for the additive white Gaussian noise (AWGN) channel. Section III establishes the spectral characteristics of the modulated signal. Section IV briefly presents the advantage provided by this modulation scheme when signaling on a Rayleigh fading channel. Section V points out an important property for coherent transmitter-receiver implementation.

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II. Signal Structure

Let the ith signal from a set of M signals be given as

$$s_i(t) = A\sum_{n=1}^{N}\cos(2\pi f_n t + \phi_{in}),$$
$$0 \le t \le T, \quad i = 1, 2, \cdots, M, \tag{1}$$

where ϕ_{in} can be any of p equally spaced phase shifts chosen from the set $\{2\pi k/p\}$ $(k = 0, 1, \cdots, p-1)$. For a fixed value of p, a complete phase code gives rise to $M = p^N$ signals. Focusing on $N = 2$, defining $\Delta f = f_2 - f_1$, and assuming $f_1, f_2 \gg 1/T$, the signal energy is given by

$$E_i = \int_0^T s_i^2(t)dt \approx A^2T\left[1 + \frac{\sin\theta}{\theta}\cos(\theta + \phi_{i2} - \phi_{i1})\right], \tag{2}$$

where $\theta = \pi T\Delta f = \pi h$. For $h = 1$, the energy $E_i = E$ is independent of i, and the correlation coefficient between the ith and jth signals can be shown to be

$$p_{ij} = \frac{1}{E}\int_0^T s_i(t)s_j(t)dt/$$
$$\approx \frac{\cos(\phi_{j1} - \phi_{i1}) + \cos(\phi_{j2} - \phi_{i2})}{2}. \tag{3}$$

We restrict our attention to symmetric phase codes that offer attractive trade-offs versus existing modulation systems. A phase code is said to be symmetric whenever the probability of a symbol error, conditioned on a particular signal being transmitted, is the same independent of the signal chosen. Such codes possess attractive features for implementation. In addition, when a maximum likelihood receiver is employed, the average symbol error probability is independent of the distribution of prior probabilities for the transmitted signals [4].

A. Complete Phase Codes

A complete phase code includes all phase combinations. Such codes will be denoted as p-FPSK. Here a symmetric M-ary set of equal energy signals is realized. The symbol error probability in AWGN can be specified with the union upper bound as

$$P_e \le \sum_{j=1, j\ne i}^{M} Q\left(\sqrt{\gamma_s(1 - p_{ij})}\right) \qquad \text{for any } i, \tag{4}$$

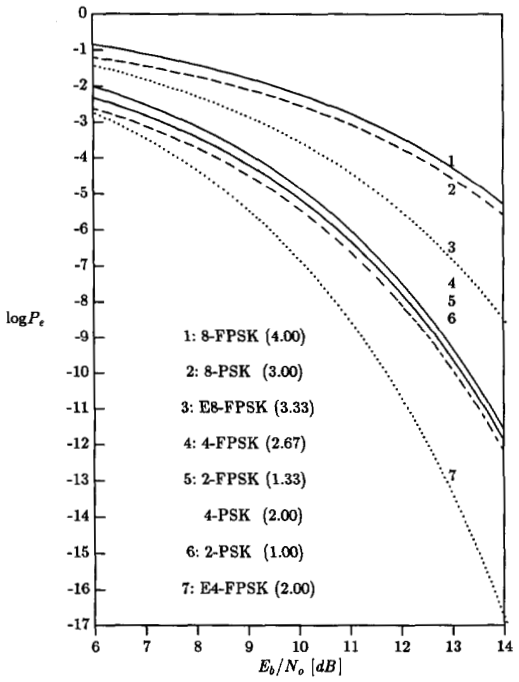

Fig. 1. FPSK modulation performance for AWGN channel.

where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2} dt,$$

and $\gamma_s = K(E_b/N_0)$, with K the number of encoded bits and E_b/N_0 the bit energy-to-noise power spectral density ratio, This upper bound is shown in Fig. 1 for complete phase codes when $p = 2$, 4, and 8 (curves 5, 4, and 1, respectively). The spectral efficiency (the bit rate to bandwidth ratio, R/W, in bits per second/hertz) of each modulation scheme (discussed in Section III) is enclosed in parenthesis in the figure. The performance is found to be degraded by 0.3 dB with respect to their two dimensional (2-D) counterparts (p-PSK) at a symbol error probability of 10^{-5} with an improvement of 33% in spectral efficiency.

B. Expurgated Phase Codes

The performance of FPSK modulation can be improved, at the expense of lower spectral efficiency, by expurgating the complete codes. Two such codes are considered, denoted by Ep-FPSK, for $p = 4$ and 8. They are suitable for encoding $K = 3$ and 5 bits, respectively.

1) E_4-FPSK for $K = 3$ bits: The signal constellation for this case is shown in Fig. 2, where $r = \sqrt{E/2}$ and each phase shift is abbreviated by an integer n_j $(j = 1, 2)$ chosen from the set $\{0, 1, \cdots, p-1\}$. Using the notation $s_i : (n_1, n_2)$, the eight signals are given by the following combinations:

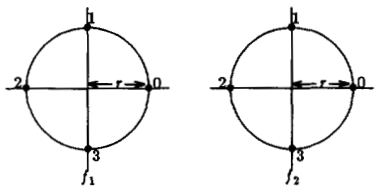

Fig. 2. Signal constellation for 4-FPSK modulation.

$$\begin{array}{ll} s_1 : (0,0) & s_5 : (0,2) \\ s_2 : (1,1) & s_6 : (1,3) \\ s_3 : (2,2) & s_7 : (2,0) \\ s_4 : (3,3) & s_8 : (3,1). \end{array}$$

These combinations were selected to minimize the maximum correlation between any pair of signals. This signal set achieves biorthogonal placement of the eight signals in four dimensions, and is therefore optimum in the sense of lowest error probability. Its performance is shown in Fig. 1 (curve 7) and is found to exceed the performance of 4-PSK (at $P_e = 10^{-5}$) by 1.3 dB with the same spectral efficiency, namely, 2(b/s)/Hz. Both E4-FPSK and 8-PSK can encode 3 bits, with the latter being 50% more spectrally efficient than the former. However, at $P_e = 10^{-5}$, E4-FPSK outperforms 8-PSK by about 5 dB.

2) E8-FPSK for $K = 5$ bits: The signal constellation for the complete 8-FPSK set is shown in Fig. 3, where r has the same value as given earlier. When half the total number of signals are deleted, but symmetry is maintained, the E8-FPSK signal set is realized. Using the same notation and expurgation criterion as given earlier, the 32 signals in E8-FPSK are given as follows:

$$\begin{array}{llll} s_1 : (0,0) & s_9 : (0,4) & s_{17} : (0,2) & s_{25} : (0,6) \\ s_2 : (1,1) & s_{10} : (1,5) & s_{18} : (1,3) & s_{26} : (1,7) \\ s_3 : (2,2) & s_{11} : (2,6) & s_{19} : (2,4) & s_{27} : (2,0) \\ s_4 : (3,3) & s_{12} : (3,7) & s_{20} : (3,5) & s_{28} : (3,1) \\ s_5 : (4,4) & s_{13} : (4,0) & s_{21} : (4,6) & s_{29} : (4,2) \\ s_6 : (5,5) & s_{14} : (5,1) & s_{22} : (5,7) & s_{30} : (5,3) \\ s_7 : (6,6) & s_{15} : (6,2) & s_{23} : (6,0) & s_{31} : (6,4) \\ s_8 : (7,7) & s_{16} : (7,3) & s_{24} : (7,1) & s_{32} : (7,5). \end{array}$$

The performance of this signal set is given in Fig. 1 (curve 3). It exhibits an improvement of about 2 dB with respect to 8-PSK for the target error rate with an 11% improvement in spectral efficiency. Fig. 4 shows a comparative performance of the modulation schemes considered for a bit error rate of 10^{-5}. The relationship between symbol and bit error rates is dependent on the mapping from bit combinations to signals. The approximation $P_b \approx 0.5P_e$ [6] has been used.

III. Spectral Efficiency

To visualize the bandwidth occupied by the 4-D modulated signal, we note that $s_i(t)$ can be represented as a single phase modulated carrier, $c_i(t)$, amplitude modulated by a baseband signal, $a_i(t)$. This is seen by rewriting (1) as

$$s_i(t) = 2Aa_i(t)c_i(t), \qquad 0 \le t \le T,$$

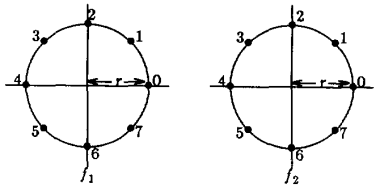

Fig. 3. Signal constellation for 8-FPSK modulation.

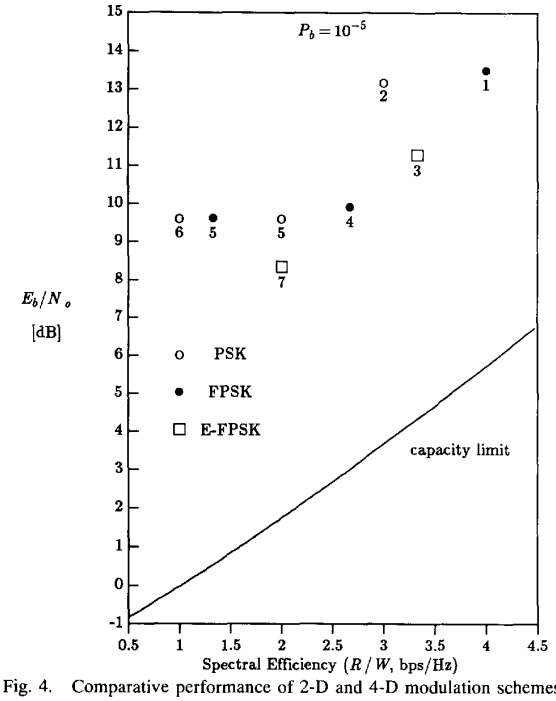

Fig. 4. Comparative performance of 2-D and 4-D modulation schemes.

$$= 2A\cos\left(2\pi\frac{\Delta f}{2}t + \frac{\phi_{i2}-\phi_{i1}}{2}\right) \cdot \cos\left(2\pi\overline{f}t + \frac{\phi_{i1}+\phi_{i2}}{2}\right), \quad 0 \le t \le T, \tag{5}$$

where $\overline{f} = 0.5(f_1 + f_2)$. With this signal description, $a_i(t)$ may be interpreted as the low-pass envelope of the carrier $c_i(t)$. Note that both the envelope and carrier phases are dependent on the index i of the transmitted signal. The bandwidth of the baseband signal, analogous to the p-PSK case, can be estimated in terms of the bandwidth of the low-frequency envelope that is time limited from 0 to T. The normalized energy spectral density, or baseband spectrum, is found from the Fourier transform of $a_i(t)$ as

$$G_i(f) = \frac{|A_i(f)|^2}{T^2/4} = \operatorname{sinc}^2\left[\left(f - \frac{\Delta f}{2}\right)T\right] + \operatorname{sinc}^2\left[\left(f + \frac{\Delta f}{2}\right)T\right] + 2\operatorname{sinc}\left[\left(f - \frac{\Delta f}{2}\right)T\right]\operatorname{sinc}\left[\left(f + \frac{\Delta f}{2}\right)T\right]\cos\alpha_i, \tag{6}$$

where $\operatorname{sinc}(x) = (\sin \pi x)/\pi x$ and $\alpha_i = \pi + \phi_{i2} - \phi_{i1}$.

For $h = 1$, the baseband spectrum has a first null bandwidth $W = 1.5/T$. Since the bit rate is $R = K/T$, this gives a theoretical spectral efficiency $R/W = K/1.5$ [(b/s)/Hz]. Fig. 5 shows the normalized baseband spectrum, $G_i(f)$, for several values of α_i. For phase codes having $\alpha_i = 0$, the low-frequency envelope effects sinusoidal pulse shaping on the RF carrier yielding a spectrum identical to that of MSK modulation [5] as verified from (6).

IV. Performance on a Rayleigh Fading Channel

The performance of multiple phase diversity codes has been derived for the case of adaptive reception on a slowly fading, frequency nonselective Rayleigh fading channel [6]. In this section, we make use of these results to exploit the intrinsic diversity of two-frequency versus single-frequency phase modulation transmission.

The symbol error probability when signaling over a Rayleigh fading channel with p-PSK transmission and Lth-order diversity is given by [6]

$$P = \frac{(-1)^{L-1}(1-\mu^2)^L}{\pi(L-1)!}\left\{\frac{\partial^{L-1}}{\partial x^{L-1}}\left[\frac{1}{x-\mu^2}\left(\frac{\pi}{p}(p-1) - \frac{\mu\sin(\pi/p)}{\sqrt{x-\mu^2\cos^2(\pi/p)}}\cot^{-1}\frac{-\mu\cos(\pi/p)}{\sqrt{x-\mu^2\cos^2(\pi/p)}}\right)\right]\right\}_{x=1}, \tag{7}$$

where $\mu = \sqrt{\overline{\gamma}_c/(\overline{\gamma}_c + 1)}$ for coherent detection and $\overline{\gamma}_c = (K/L)(\overline{E}_b/N_0)$, with $\overline{E}_b$ representing the average bit energy. Although this result has been developed for diversity codes (i.e., codes having $\phi_{i2} = \phi_{i1}$), it also applies to biorthogonal codes, such as the E4-FPSK, when the receiver is provided with the necessary information to decode each signal in a manner similar to a maximum likelihood detector. This is accomplished by defining

$$h_{in} = e^{-j\phi_{in}}, \qquad n = 1, 2, \tag{8}$$

as *phase aligning coefficients* that are used to modify the matched filters at each of the frequencies f_1 and f_2. Similar to the approach in [6], the operation of phase alignment can be combined with the process of compensating for the fading channel's gain and phase shift.

Fig. 6 shows the performance on a Rayleigh fading channel for 2- and 4-PSK when $L = 1$ (no diversity), and when $L = 2$. Also included is the performance of the E4-FPSK modulation scheme discussed in Section II. The E4-FPSK scheme outperforms 2- and 4-PSK (each with second-order diversity) and has superior spectral efficiency.

V. Coherent Implementation

From (5), it is evident that FPSK signals do not, in general, possess a constant envelope. The transmitted phase sequence can be estimated by phase-locking to the individual frequencies, f_1 and f_2. A suboptimum approach to estimate the phase sequence (ϕ_{i1}, ϕ_{i2}) may be implemented by deriving the phase shifts of $c_i(t)$ and $a_i(t)$. These phase shifts can then be added and subtracted to provide estimates of ϕ_{i2}

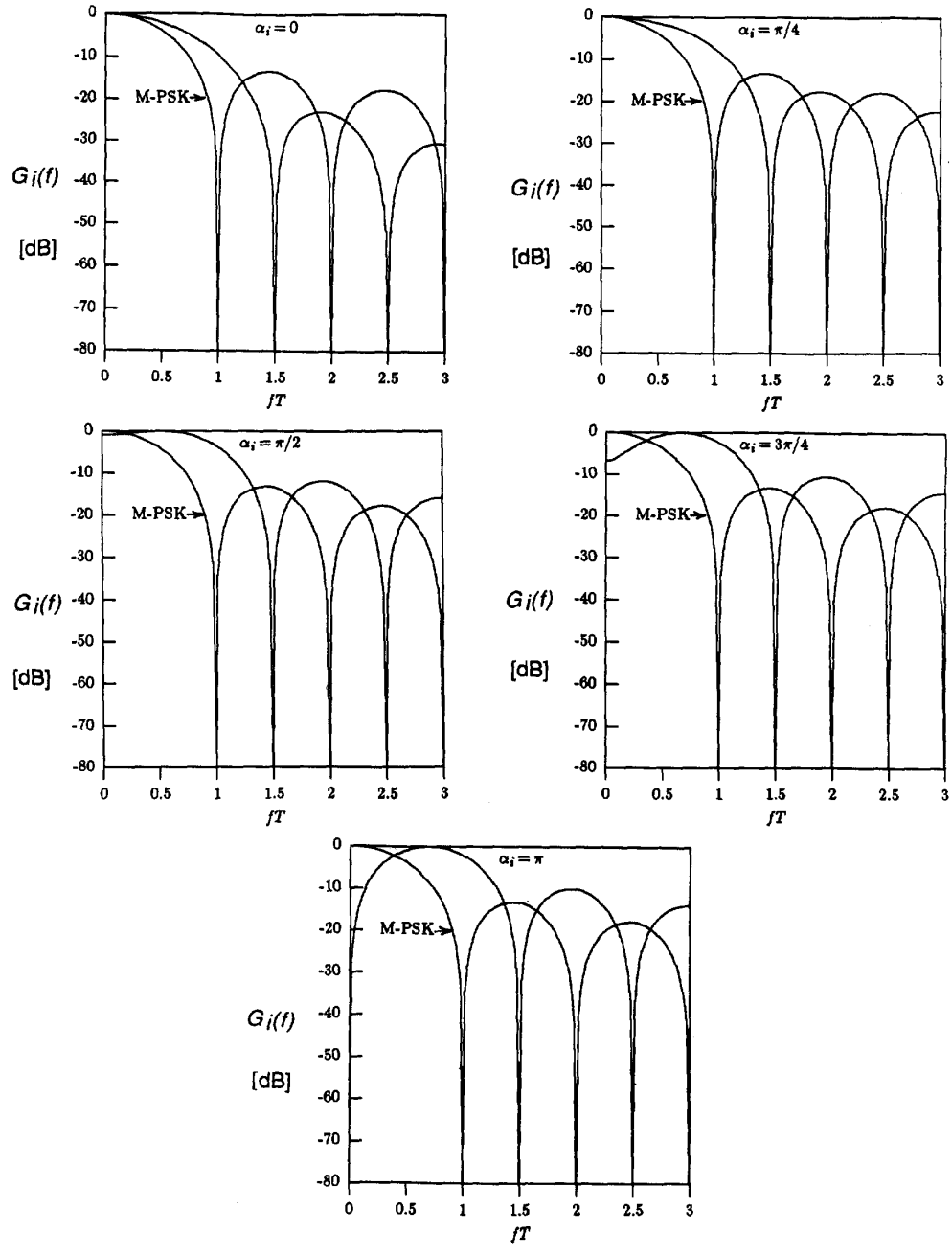

Fig. 5. Normalized baseband spectrum, $G_i(f)$ (dB), for different α_i.

and ϕ_{i1}, respectively. This second approach may offer an advantage when the two frequencies are considerably larger than their frequency separation (i.e. when phase acquisition of the individual frequencies difficult); and when the phase of the low-frequency envelope can be estimated reliably. Finally, it should be noted that a coherent transmitter/receiver implementation can be realized with a single RF carrier whose frequency equals the mean of f_1 and f_2, provided that the phase of the low-frequency envelope can be determined.

VI. Conclusions

The performance of a hybrid frequency and phase-shift-keying (FPSK) modulation scheme suitable for spectrally efficient applications has been determined. The modulation schemes considered offer attractive performance tradeoffs with respect to M-PSK modulation for the Gaussian channel, and an intrinsic degree of diversity which translates into considerable advantage when signaling on a fading channel.

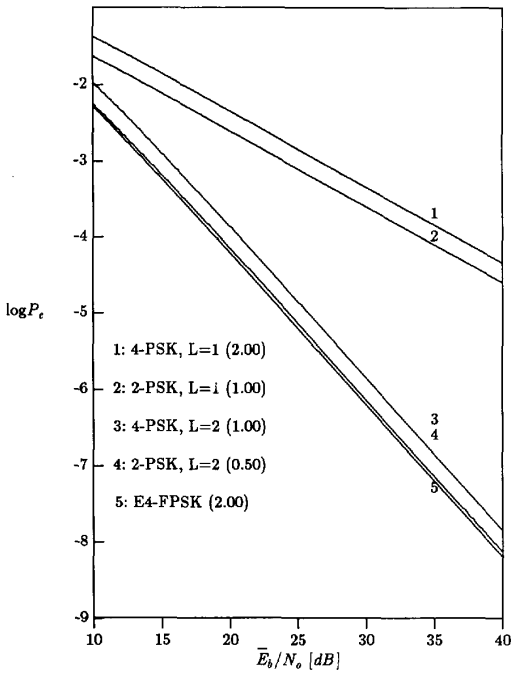

Fig. 6. Performance comparison for Rayleigh fading channel.

A study of the performance of a coded phase- and frequency-modulated system can be found in [7]. More recently, the uncoded and coded performance of a four-dimensional modulation scheme similar to the one addressed herein has been examined [8], [9], where a pair of orthogonal pulse shapes were used to modulate two phase orthogonal carriers.

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