Modified Modulation Formats using Time-Varying Phase Functions

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***Abstract*— New modulation formats are presented that increase the transmission rate over that of conventional systems, without degrading the bit error rate (BER) and with minimal bandwidth variation. A time-varying function called an "extra phase variation function" (EPVF) is added to the discrete phase of conventional modulation formats such as M-PSK and QAM to transmit additional data bits. A receiver configuration is presented that allows the decoding of information represented by the discrete and extra phases. It is shown that in an additive white Gaussian noise (AWGN) channel, the BER performance of the bits carried by the discrete phase and the extra phase in the modified format improves over the BER of conventional modulation formats.**

***Index Terms*— Bit-error rate, modulation, spectral efficiency.**

I. Introduction

EFFICIENT use of bandwidth and power to achieve higher transmission rates is a main concern in the telecommunication industry. Data intensive applications running on wireless systems require a much higher rate than before, while maintaining the performance and bandwidth. The goal of a modulation format is to transmit a message signal through a radio channel, with the lowest probability of error, minimum average power, and the highest bit rate possible with a given bandwidth. Most systems can be classified as either bandwidth limited or power limited. In bandwidth limited systems, spectrally efficient modulation techniques are used to conserve bandwidth at the expense of power and/or performance, whereas in power limited systems, power efficient modulation techniques are used to conserve power at the expense of bandwidth and/or performance [1], [2].

For an M-ary modulation system, the information rate can be expressed as $R = \log_2 M/T_s$ bits/second where M is the number of equally likely signal waveforms transmitted during each symbol interval, T_s. If M-ary Frequency Shift Keying (FSK) is used, the bandwidth required is increased for $M > 2$. To avoid the bandwidth expansion, M-ary PSK could be used, which would lead to performance degradation versus M-ary FSK for increasing M, with the same power [1]–[3]. An optimal tradeoff between BER performance and bandwidth is studied in [4]–[7] that uses families of phase smoothing functions in Continuous Phase Modulation (CPM) for spectral shaping. The data rate however does not improve. A joint frequency-phase modulation has also been investigated in [8]–[10], that seeks to achieve a good tradeoff between spectral efficiency and performance by combining FSK and PSK formats. In general, the bandwidth efficiency obtained in this case is less than that of the new modified modulation format proposed in this paper.

In this paper, a new modulation method is persented that increases the data transmission rate over that of conventional M-ary formats (M-PSK or M-QAM) by mapping additional information bits to extra phase functions, which are added to the conventional discrete phase. It is shown that there are minimal bandwidth variations compared to conventional modulation formats, and the performance of the conventional and the additional information bits in an AWGN channel is better than that of the original M-ary format. Increasing the number of EPVFs can increase the data transmission capability of the modified modulated signal. Phase continuity can be imposed on the system to control bandwidth by adding memory. This will further improve the spectral performance of the system.

A receiver is designed for decoding the information carried by the M discrete phases and the additional information carried by the extra phase function. Section II of this paper introduces the signal model. Section III presents the receiver design and some suitable EPVFs based on the constraints on the signal, required to demodulate and detect the data. This section also compares the BER performance of the extra information bits and the basic information bits in the modified modulation format system with systems using conventional M-PSK and M-QAM formats. Section IV compares the bandwidth efficiencies of the modified modulation format with conventional formats. Conclusions are given in Section V.

II. Signal Model

The general mathematical representation of a modified modulated QAM signal is:

$$\begin{aligned} s(t) &= a_i g(t)\cos(2\pi f_0 t + \theta_i + \varphi_j(t)) \\ &\quad 0 \le t \le T_s, \quad 1 \le i \le M, \quad 1 \le j \le M_{ex} \end{aligned} \tag{1}$$

where f_0 is the carrier frequency, $g(t)$ is a pulse shape of duration T_s. The conventional QAM data is contained in the discrete amplitude values $a_i \in \mathbb{R}$ and the corresponding phase values $\theta_i \in [\frac{2i\pi}{M}]$. The time-varying phase $\phi_j(t)$ represents the additional data. For the case of unshaped, modified M-ary PSK with an energy per symbol E_s, (1) becomes:

$$s(t) = \sqrt{\frac{2E_s}{T_s}}\cos(2\pi f_0 t + \theta_i + \varphi_j(t)) \tag{2}$$

where each of the M values of θ_i carry $k = \log_2 M$ basic bits of information, and each of the M_{ex} EPVFs $\varphi_j(t)$ carry $k_{ex} = \log_2 M_{ex}$ extra bits of information.

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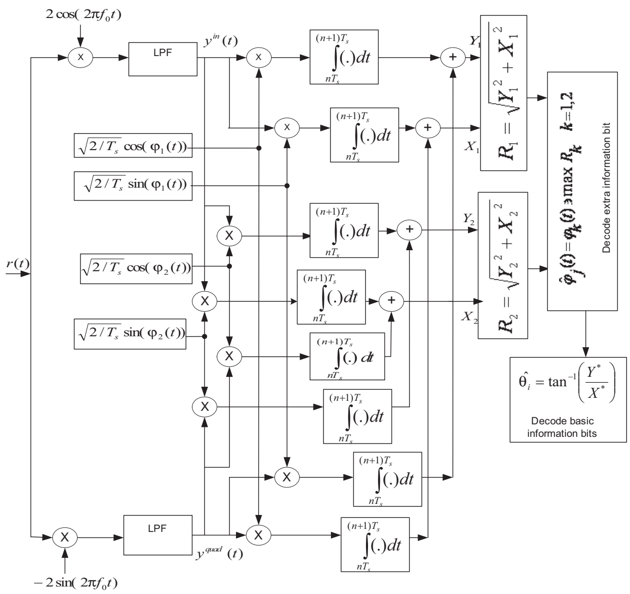

Fig. 1. Receiver structure.

TABLE I
SOLUTIONS OF DIFFERENT ODD EPVFS

	Formats of the EPVFs	Example of the possible solutions
Example 1	$\varphi_1(t, a_1) = a_1 \sin(2\pi t/T_s)$	$\varphi_1(t, a_1) = 1.202 \sin(2\pi t/T_s)$
	$\varphi_2(t, a_2) = a_2 \sin(2\pi t/T_s)$	$\varphi_2(t, a_2) = -1.202 \sin(2\pi t/T_s)$
Example 2	$\varphi_1(t, a_1) = a_1(2\pi t/T_s)$	$\varphi_1(t, a_1) = 0.491(2\pi t/T_s)$
	$\varphi_2(t, a_2) = a_2(2\pi t/T_s)$	$\varphi_2(t, a_2) = -0.491(2\pi t/T_s)$

III. RECEIVER DESIGN AND ERROR RATE PERFORMANCE

We can write (2) as:

$$\sqrt{\frac{2E_s}{T_s}}((\cos(\varphi_j(t))\cos\theta_i - \sin(\varphi_j(t))\sin\theta_i))\cos(2\pi f_0 t) - \sqrt{\frac{2E_s}{T_s}}((\cos(\varphi_j(t))\sin\theta_i + \sin(\varphi_j(t))\cos\theta_i))\sin(2\pi f_0 t) \tag{3}$$

Synchronous demodulation of (3) with $2\cos(2\pi f_0 t)$ and a low pass filter yields the in-phase term:

$$y^{in}(t) = \sqrt{\frac{2E_s}{T_s}}\,(\cos(\varphi_j(t))\cos\theta_i - \sin(\varphi_j(t))\sin(\theta_i))) \tag{4}$$

Similarly, demodulation of (3) with $-2\sin(2\pi f_0 t)$ produces a quadrature term:

$$y^{quad}(t) = \sqrt{\frac{2E_s}{T_s}}\,(\cos(\varphi_j(t))\cos\theta_i + \sin(\varphi_j(t))\sin\theta_i)) \tag{5}$$

It has been shown in [11] that the extra data can be readily recovered if the EPVFs are chosen to satisfy the following condition:

$$\int_0^{T_s} \sin(\varphi_j(t) - \varphi_k(t)))dt = 0 \tag{6}$$

Under the condition (6), the sum of $y^{in}(t)$ correlated with $\sqrt{2/T_s}\cos(\varphi_k(t))$ and $y^{quad}(t)$ correlated with $\sqrt{2/T_s}\sin(\varphi_k(t))$ results in an in-phase variable X defined as $X = X(\theta_i, \varphi_j(t), \varphi_k(t))$, given by:

$$X = \left(\frac{\sqrt{E_s}}{T_s}\right)\cos\theta_i \int_0^{T_s} \cos(\varphi_j(t) - \varphi_k(t))dt \tag{7}$$

Similarly, under the condition in (6), the difference between $y^{quad}(t)$ correlated with $\sqrt{2/T_s}\cos(\varphi_k(t))$ and $y^{in}(t)$ correlated with $\sqrt{2/T_s}\sin(\varphi_k(t))$ generates a quadrature variable Y defined as $Y = Y(\theta_i, \varphi_j(t), \varphi_k(t))$, given by:

$$Y = \left(\frac{\sqrt{E_s}}{T_s}\right)\sin\theta_i \int_0^{T_s} \cos(\varphi_j(t) - \varphi_k(t))dt \tag{8}$$

The in-phase and quadrature variables, X and Y, form the components of a decision metric that can be used to estimate the EPVFs. Let us define $R(\varphi_j, \varphi_k)$ as:

$$\begin{aligned} R(\varphi_j, \varphi_k) &= \sqrt{X^2 + Y_2} \\ &= \left(\frac{\sqrt{E_s}}{T_s}\right)\int_0^{T_s} \cos(\varphi_j(t) - \varphi_k(t))dt \end{aligned} \tag{9}$$

The decision metric should ideally be zero when $j \neq k$ and $\sqrt{E_s}$ when $j = k$. The EPVFs will be chosen to guarantee this condition in $R(\varphi_j, \varphi_k)$. Thus, in the presence of zero mean AWGN, the decision rule for equally likely EPVFs will be:

$$\hat{\varphi}_j(t) = \varphi_k(t) \ni maxR(\varphi_j, \varphi_k), \quad 1 \leq k \leq M_{ex} \tag{10}$$

Following the detection of $\varphi_j(t)$, the in-phase and quadrature variables are evaluated to be:

$$X^* = \sqrt{E_s}\cos(\theta_i) \tag{11}$$

and

$$Y^* = \sqrt{E_s}\sin(\theta_i) \tag{12}$$

The discrete phase term is then estimated as:

$$\hat{\theta}_i = \arctan(Y^*/X^*) \tag{13}$$

The receiver structure consists of: (a) synchronous demodulation (in-phase and quadrature), (b) a bank of correlators to generate X and Y for each $\varphi_k(t)$, (c) a decision metric computation for $\hat{\varphi}_j(t)$ and (d) discrete phase estimation for $\hat{\theta}_i$. A block diagram of a modified modulation format receiver with $M_{ex} = 2$ is given in Fig. 1.

A. EPVF Example

The design of a modified modulation format signal, $s(t)$, requires a set of EPVFs, $\varphi_j(t)$, that satisfy (6). For the case of $M_{ex} = 2$, $\varphi_1(t)$ and $\varphi_2(t)$ must be defined. It can be observed, that for (6) to be satisfied, the EPVFs $\varphi_j(t)$ need to be odd functions. A simple odd EPVF is a sinusoid. To transmit an additional bit per symbol, consider the following EPVF pair:

$$\varphi_1(t, a_1) = a_1 \sin(2\pi t/T_s) \tag{14}$$

$$\varphi_2(t, a_2) = a_2 \sin(2\pi t/T_s) \tag{15}$$

It is thus required to find the coefficient pair (a_1, a_2) such that $R^2(\varphi_1, \varphi_2) = 0$. A three-dimensional plot of $R^2(\varphi_1, \varphi_2)$ versus (a_1, a_2) reveals a contour in the $R^2(\varphi_1, \varphi_2) = 0$ plane, and many possible coefficient pairs (a_1, a_2). One possible solution is to choose the intersection of the line $a_1 + a_2 = 0$ with the contour in the $R^2(\varphi_1, \varphi_2) = 0$ plane. Two examples of possible solutions with odd EPVFs are shown in Table I.

The performance of different modulation formats is traditionally compared by examining the BER as a function of the ratio E_b/N_0 [1], [2], [12], [13] where E_b is the energy per bit and N_0 is the one-sided white noise power spectral density (PSD). In the case of the modified modulation format, the relationship between bit energy and symbol energy is $E_b = E_s/(k + k_{ex})$. Thus the symbol energy for the modified modulation format can be increased by a factor of $1 + k_{ex}/k$ to keep the energy per bit the same as in the conventional modulation format when comparing the BER performances of the two systems. To evaluate the BER performance, a Monte Carlo simulation was performed with AWGN for an 8-PSK and 4-PSK modified modulation formats with $M_{ex} = 2$. The results are shown in Fig. 2. A better performance is clearly observed on both the basic and extra bits of the modified 8-PSK format. A similar observation carries over to M-PSK for $M > 8$ [11]. For the 4-PSK format, a better performance is observed on the extra bit after $E_b/N_0 = 3dB$. There is an accompanying improvement in the transmission rate for all M-ary formats. The percentage increase in rates of transmission for modified 8-PSK and modified 4-PSK are 33 percent and 50 percent respectively. Additional simulations [11] show that if the symbol energy E_s is kept the same for the conventional and modified formats ($M \geq 8$), the performance of the basic bits remains the same as in the conventional format, while there is a performance improvement on the extra bit.

BER performance was also investigated for a conventional rectangular 16-QAM and its modified form with $M_{ex} = 2$.

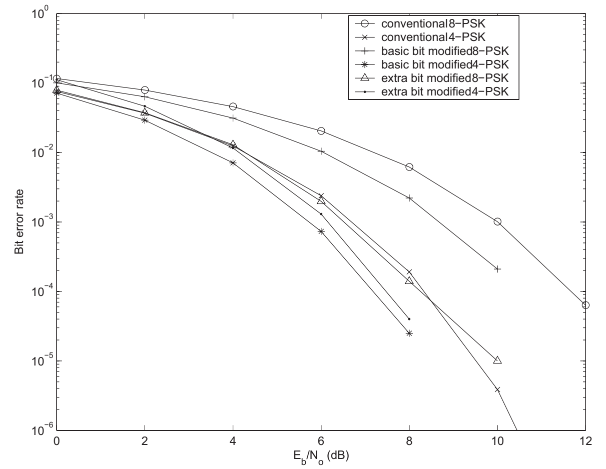

Fig. 2. Bit error rate comparison of conventional 8-PSK and 4-PSK with their corresponding modified formats, with one extra bit.

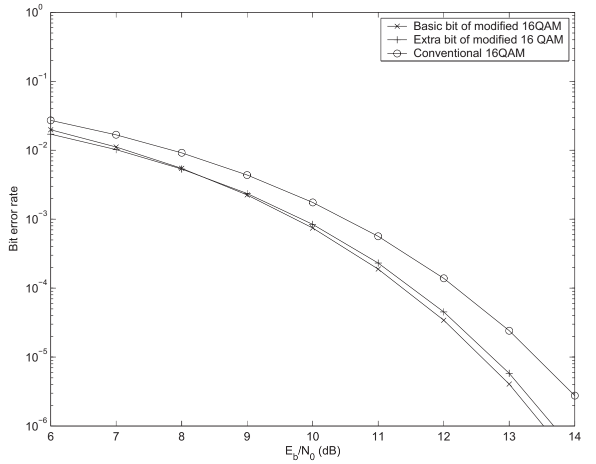

Fig. 3. Bit error rate comparison of conventional 16-QAM and modified 16-QAM with one extra bit.

Fig. 3 shows the BER performance of a modified 16-QAM format compared to the conventional 16-QAM format, where the approximate relation between bit error rate and symbol error rate has been used [1], [10]. Again, a performance improvement is observed on both the basic and extra bits. The data rate in this case improves by 25 percent. With any PSK or QAM modulation system, a much larger increase in the data rate is possible using a larger set of EPVFs.

IV. BANDWIDTH EFFICIENCY OF THE MODIFIED PSK FORMAT

The bandwidth efficiency of a conventional modulation system can be defined as k/T_sB, where B is the bandwidth of the modulated signal defined in terms of the fractional out of band power containement. For a modified modulation system, this efficiency is $(k+k_{ex})/T_sB$. The PSD of the conventional and modified 8-PSK and 4-PSK was computed numerically using Welch's periodogram averaging method [14]–[16] with $M_{ex} = 2$ and the EPVFs in Example 1, Table I. In all cases,

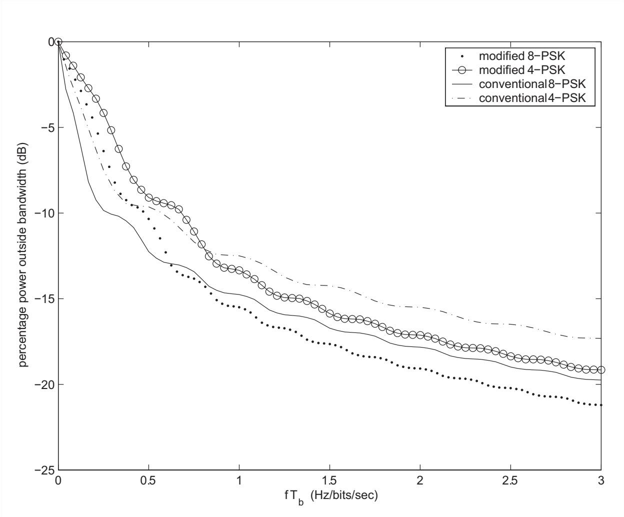

Fig. 4. Fractional out of band power for conventional 8-PSK and 4-PSK compared to modified 8-PSK and 4-PSK with EPVF in Example 1 in Table 1.

the fractional out of band power bandwidth was evaluated. Fig. 4 shows the fractional out of band power versus the normalized bandwidth fT_b for modified 4-PSK and 8-PSK formats compared to the conventional formats. Observe that the modified formats have a smaller bandwidth for fT_b greater than 0.65 for 8-PSK and for fT_b greater than 0.85 for 4-PSK. Thus, the bandwidth efficiency for the modified modulated system will be larger than the conventional systems because of both additional bits and smaller bandwidth. Again, a larger set of EPVFs is expected to further improve the bandwidth efficiency.

V. Conclusion

New modulation formats to increase the transmission rate and improve the performance with minimal bandwidth variations are studied.A receiver structure is proposed for detecting the basic and extra information contained in the discrete phase and the extra phase function respectively. Examples of pairs of EPVFs were given that satisfied the demodulator constraints and provided optimum detection of the time-varying phase function. A performance comparison was made between the modified and conventional modulation systems in an AWGN channel. The performance of the extra bit and the basic bits improved in 4-PSK, 8-PSK, and 16-QAM. The fractional out of band power was compared with a conventional system and no significant bandwidth variations were observed, enabling an increase in the system bandwidth efficiency. The improvements are obtained at the expense of a complex receiver structure. Work on reducing the receiver complexity and analyzing the system performance in a fading channel is in progress.

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