Energy-Efficient Capacity Optimization in Wireless Networks

Lu Liu*, Xianghui Cao*, Yu Cheng*, Lili Du†, Wei Song‡ and Yu Wang§
*Department of Electrical and Computer Engineering, Illinois Institute of Technology, USA
Email: lliu41@hawk.iit.edu; xh.cao@ieee.org; cheng@iit.edu
†Department of Civil, Architectural and Environmental Engineering, Illinois Institute of Technology, USA
Email: ldu3@iit.edu
‡Faculty of Computer Science, University of New Brunswick, Canada
Email: wsong@unb.ca
§Department of Computer Science, University of North Carolina at Charlotte, USA
Email: yu.wang@uncc.edu

***Abstract*—We study how to achieve optimal network capacity in the most energy-efficient manner over a general large-scale wireless network, say, a multi-hop multi-radio multi-channel (MR-MC) network. We develop a multi-objective optimization framework for computing the resource allocation that leads to optimal network capacity with minimal energy consumption. Our framework is based on a linear programming multi-commodity flow (MCF) formulation augmented with scheduling constraints over multi-dimensional conflict graph (MDCG). The optimization problem however involves finding all independent sets (ISs), which is NP-hard in general. Novel delayed column generation (DCG) based algorithms are developed to effectively solve the optimization problem. The DCG-based algorithms have significant advantages of low computation overhead and achieving high energy efficiency, compared to the common heuristic algorithm that randomly searches a large number of ISs to use. Extensive numerical results demonstrate the energy efficiency improvement by the proposed energy-efficient optimization techniques, over a wide range of networking scenarios.**

***Index Terms*—Multi-radio multi-channel network; capacity optimization; energy efficiency; multi-objective optimization**

I. Introduction

In large-scale multi-hop wireless networks, simultaneously achieving capacity (or throughput) optimization and energy efficiency is a challenging issue. Many existing studies in the context of either traditional single-radio single-channel (SR-SC) [1]–[4] or multi-radio multi-channel (MR-MC) networks [5]–[8] often focus on network capacity; however, the important problem of energy-efficient capacity optimization over a generic MR-MC network[1] has not been well investigated yet. Being able to simultaneously exploit multiple channels through different radio interfaces, an MR-MC network can have significantly higher capacity than an SR-SC network [6], [7]. However, in MR-MC networks with increased system dimensions, energy-efficient capacity optimization requires jointly considering the coupling issues of routing, link scheduling and channel/radio assignment.

A fundamental issue in multi-hop wireless networks is the interference due to co-channel transmissions. In order to model the interference among different wireless links, the conflict graph (or contention graph) tool is often adopted, where the links that can transmit simultaneously without mutually causing interference construct an independent set (IS) over the graph. Augmented with constraints derived from the conflict graph, the main-thread approach for wireless network capacity optimization is to apply a linear programming (LP) multi-commodity flow (MCF) formulation [1]. Such a formulation has been extended to general MR-MC networks based on the recently developed multi-dimensional conflict graph (MDCG) [7], [10]. The convex hull over all the possible ISs defines the space of resource allocation. The optimal resource allocation for capacity indicated by the MCF solution can be expressed as an *IS-based scheduling* problem: the ISs take turns to access the channels for data transmission, with the proportion of transmission time of each IS determined by the MCF solution.

Maximizing capacity and minimizing energy consumption are often contradicting objectives in wireless networks and thus need careful investigation. In our primary work on MR-MC networks towards joint optimization of these two aspects, we show that a significant amount of energy can be saved by trading only a small portion of capacity [16]. In this paper, the optimization formulation in [16] is enhanced to a multi-objective optimization framework to jointly optimize network capacity and energy efficiency, where we define energy efficiency as the network throughput per unit of energy consumption, or equivalently, the ratio between achieved capacity and the energy consumption. The multi-objective framework consists of two steps. In the first step, the MCF problem is solved over the MDCG to obtain the optimal capacity. In the second step, a multi-objective optimization problem is formulated, which is solved by first transforming the capacity objective into constraint and then minimizing energy consumption over the whole network.

Considering the exponentially many possible ISs, the resource allocation space can be significantly large. Finding all possible ISs is an NP-hard problem in general [9]. A common approach in practice is to randomly search a reasonable number of ISs, which form a subspace of the entire solution

[1] The SR-SC network model can be viewed as a special MR-MC model.

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space. The searched ISs are then used in the MCF problem to obtain an approximate optimal solution [1], [16]. Such approximation over a subspace further motivates the search of an energy-efficient solution in the sense that an IS-based scheduling over a different subspace may lead to a higher network capacity but lower energy consumption.

The random search (RS) algorithm, however, suffers from the large computation overhead in searching the ISs and solving the large-scale optimization problem. It has been shown that RS may even miss some critical ISs that are of structure-level importance towards the optimal scheduling [6]. In this paper, we propose to use the delayed column generation (DCG) method [11] to solve the two LP problems formulated within the multi-objective optimization framework. With the DCG method, an LP problem is first solved to obtain a feasible solution over a subspace; then a new column that can lead to an improved solution (e.g., a larger capacity or a lower energy consumption) is searched to enter the subspace. The steps are run iteratively till converging to the optimal solution. We propose two DCG based algorithms in solving the multi-objective optimization problem. The first one, termed as *DCG-S* algorithm, is to apply DCG in solving the capacity optimization problem (i.e., step 1 in the proposed framework) with the energy efficiency problem (i.e., step 2 in the proposed framework) solved over the IS subspace obtained in step 1. The second one, termed as *DCG-W* algorithm, applies DCG in both steps where the step-2 problem is independently solved over the entire IS space. We show that the DCG-W algorithm can achieve an energy efficiency higher than the DCG-S with a slightly increased computation overhead.

There are many studies on energy-efficient networking in the literature. Most of the existing works focus on energy-efficient protocols, such as medium access control (MAC) and routing protocols, in the context of single-hop or SR-SC networks [12], [13]. This paper systematically studies how to calculate the energy-efficient optimal network capacity and the associated resource allocation in a general context of large-scale multi-hop MR-MC wireless networks. Major contributions of this paper can be summarized as follows.

1) We develop a multi-objective optimization framework for energy-efficient network capacity optimization based on the MCF formulation augmented with the tool of MDCG. The multi-objective optimization framework based on the MDCG can jointly generate cross-layer optimal resource allocation of routing, link scheduling, and channel/radio assignment.
2) Novel DCG-based algorithms are proposed to effectively solve the optimization problem. We show that the proposed DCG algorithms have the advantages over the RS algorithm [16] in terms of lower computation overhead as well as higher network capacity and better energy efficiency.
3) Extensive numerical results are presented to demonstrate the energy efficiency gain by the proposed algorithms, over a wide range of scenarios with different network size, different number of channels or radios. The performance and computation overhead of the RS algorithm and the proposed DCG based algorithms are also evaluated and compared.

The remainder of this paper is organized as follows. Section II reviews more related work. Section III describes the system model. Section IV presents the multi-objective optimization framework. Section V develops the DCG-based algorithms. Numerical results are presented in Section VI. Finally, Section VII gives the conclusion remarks.

II. Related Work

Energy efficient networking has gained significant attention in the development of wireless networks. Many existing studies focus on the design of energy efficient protocols in SR-SC networks. For example, energy efficient MAC protocols for WLANs are studied in [14] and [15], where only single-hop scenario is considered. Power saving techniques are also proposed to prolong the lifetime of wireless sensor networks such as in [17], which focuses on listening protocol design and in [18], which concentrates on routing layer. In this paper, we propose an energy efficiency optimization framework that can jointly generate cross-layer optimal resource allocation in general MR-MC networks.

Since the resource allocation problem is characterized by two conflicting objectives of capacity and energy, multi-objective optimization is adopted for the problem formulation. Many methods have been developed to solve multi-objective optimization problems. Evolutionary approaches are used in [19] and [20], where genetic algorithms are applied for the network optimization. However, such methods run iteratively and the computation time for getting an optimal solution can be very long. With the advantage of linearity in our problem, some lightweight methods with less computation cost can be used, such as weighting method and ϵ-constraint method [21]. The weighting method generates a weighted sum of all the objective functions as the new objective. However, the weighted sum of the two objectives in our problem may not have clear physical meanings and the weights are difficult to decide beforehand. In this paper, a ϵ-constraint [21] based method is applied for the problem formulation.

Our work also exploits the MR-MC technique in formulating the generic optimization model. The MCF formulation is implemented in MR-MC scenarios based on MDCG developed in [7]. The formulated multi-objective optimization problem is of extremely large scale since the exponentially many ISs will induce large number of columns in the constraint matrix. We introduce the DCG method to iteratively search profitable columns and can efficiently solve the problem. The DCG method has been applied for scheduling [22] and resource optimization [23], but their studies are limited to special scenarios.

III. System Model

In this section, we introduce a generic system model based on MR-MC networks. An MR-MC wireless network can be represented by a connected graph $\mathcal{G}(\mathcal{N}, \mathcal{L})$ along with an

MDCG as defined latter in this section, where $\mathcal{N}$ and $\mathcal{L}$ denote the set of nodes and set of wireless links in the network, respectively. Two nodes u and v form a directed link l_{uv} if v lies in the transmission range of u. Let $\mathcal{C}$ denote the set of available channels of the network, and $\mathcal{R}_u$ denote the set of radio interfaces equipped on node u. Note that an interface of a node can only tune to one channel at one time, but can be switched to other channels at different time. The capacity of the wireless link l_{uv} on a channel c is denoted as w_{uv}^c.

In order to describe the complex co-channel interferences as well as the aforementioned radio interface conflicts in MR-MC networks, we adopt the conflict graph approach, which is a common way for interference representation in SR-SC networks. An MR-MC network can be interpreted as a multi-dimensional resource space and the dimensions are defined by radios, links and channels [10]. The resource points can be represented as radio-link-channel tuples. Each tuple is represented as $((u,v),(r_u,r_v),c)$, which indicates that the link (u,v), with node u using interface r_u and node v using r_v, operates on channel c. In this way, a link (u,v) in $\mathcal{L}$ can be mapped to multiple tuples, specifically, $|\mathcal{R}_u|\times|\mathcal{R}_v|\times|\mathcal{C}|$ tuples where $|\cdot|$ denotes the cardinality of a set. By considering all available radios and channels in the network, we can obtain all the tuples which then form the vertices in the MDCG.

A conflict graph is constructed based on the protocol interference model where the conflict relationship among links is defined by their interference ranges. For ease of exposition, we assume all the nodes have the same interference range[2] such that two nodes interfere with each other if they are within each other's interference range. If two links have interfering nodes, they conflict with each other. Under protocol interference model, a link can perform a successful transmission only if both its sender and receiver are free of interference. We extend the protocol interference model to accommodate MR-MC networks and determine the conflict relationships by the following events [5]; if any of the two events occurs, the two tuples conflict with each other, which is represented by an edge between them in the MDCG.

1) **Interference conflict**: Two tuples are associated with nodes locating within each other's interference range according to the protocol interference model and work on the same channel;
2) **Radio conflict**: Two tuples share common radio interface at one or two nodes.

In the MDCG, an IS is a set of tuples free of mutual conflict relationships, i.e., the corresponding links in the original network with the radio and channel configurations indicated by these tuples can transmit simultaneously. A maximum independent set (MIS) over the MDCG is an IS to which adding any other tuple will violate the "independence". Notice that a tuple may belong to multiple ISs. Denote a generic IS over the MDCG as $\mathcal{I}$ and the set of all the ISs as $\mathcal{M}$.

[2]With diverse interference ranges, the interference relationship may become asymmetric and the resulting MDCG becomes directed. However, this does not affect the validity of our proposed methods.

Assume that the system operates on the basis of time slots of unit length. In a time slot, only the tuples belonging to the same IS can be turned on (only one IS can be activated). Thus, a scheduling is formed when different ISs share the time alternately for transmissions.

IV. PROBLEM FORMULATION

In this section, we formulate the MCF-based linear programming problem for solving the resource allocation and energy efficient capacity optimization in MR-MC networks. With the help of MDCG, the new formulation facilitates jointly deciding the optimal scheduling of link transmissions, channel assignment and radio interface assignment.

A. MCF Problem Formulation

The formulation of the MCF problem over a general MR-MC network has been introduced in [7] and [16]. In this sub-section, we briefly present the problem constraints. Based on them, two optimization problems will be formulated in the next sub-sections. Consider multiple commodity flows in the network with each flow specified by its source-destination pair $(s,d)\in\Omega$, where Ω is the set of all source-destination pairs. Specifically, the flow on l_{uv} associated with pair (s,d) is denoted as $f_{(s,d)}(u,v)$. Each flow has a rate requirement $r_{(s,d)}$. To avoid flow starvation and ensure fairness in the network, we use λ as *network capacity* in the sense that at least $\lambda r_{(s,d)}$ amount of throughput can be ensured for each commodity flow [6]. Therefore, $\forall(s,d)\in\Omega$, all the outgoing flows from the source s sum to the corresponding throughput of this commodity, i.e.,

$$\sum_{l_{su}\in\mathcal{L}} f_{(s,d)}(s,u)=\lambda r_{(s,d)} \tag{1}$$

where $\forall l_{uv}\in\mathcal{L}$,

$$f_{(s,d)}(u,v)\geq 0, \tag{2}$$

$$\sum_{(s,d)\in\Omega} f_{(s,d)}(u,v)\leq B_{uv}. \tag{3}$$

B_{uv} is the capacity bound of link l_{uv} which is explained latter in (8). Besides, for each node, its incoming and outgoing flows should be balanced, which results in the flow conservation constraints such that $\forall(s,d)\in\Omega$, and $\forall u\in\mathcal{N}/\{s,d\}$,

$$\sum_{v:l_{uv}\in\mathcal{L}} f_{(s,d)}(u,v)=\sum_{w:l_{wu}\in\mathcal{L}} f_{(s,d)}(w,u) \tag{4}$$

Since there is no incoming (outgoing) traffic at a source (destination) node, $\forall(s,d)\in\Omega$, we have

$$\sum_{u:l_{us}\in\mathcal{L}} f_{(s,d)}(u,s)=0, \tag{5}$$

$$\sum_{u:l_{du}\in\mathcal{L}} f_{(s,d)}(d,u)=0, \tag{6}$$

For optimal network throughput, different ISs will be turned on alternately in different slots for transmission. Therefore, the realized rate of link l_{uv} is bounded by both the physical link capacity and the portion of time that the link is scheduled. We denote the fraction of time allocated to IS $\mathcal{I}_m$ as α_m, where

$$\sum_{m=1}^{|\mathcal{M}|} \alpha_m \leq 1, \quad 0 \leq \alpha_m \leq 1, \forall m \tag{7}$$

Then, the flow bound B_{uv} on link l_{uv} is seen to be the sum of active channel capacities, i.e.,

$$B_{uv} = \sum_{m: l_{uv} \in \mathcal{I}_m} \alpha_m w_{uv}^{c(l_{uv}, \mathcal{I}_m)}, \forall l_{uv} \in \mathcal{L} \tag{8}$$

where $c(l_{uv}, \mathcal{I}_m)$ denotes the channel allocated to l_{uv} when $\mathcal{I}_m$ is scheduled.

B. Network Capacity Optimization

The capacity λ can be viewed as a measure of the network throughput, and optimizing network throughput is equivalent to maximizing λ. In fact, maximizing λ under the flow conservation constraints and scheduling constraints is a common formulation of the MCF problem. Denote this basic single-objective problem for capacity optimization as $\mathcal{P}_\mathbf{1}$:

$$\mathcal{P}_\mathbf{1}: \quad \begin{cases} \text{minimize}: & \lambda \\ \text{subject to}: & \text{constraints (1)-(8)} \end{cases} \tag{9}$$

Note that, in $\mathcal{P}_\mathbf{1}$, the variables to be determined include λ, the flows $\{f_{(s,d)}(u,v)\}$ and the scheduling parameters $\{\alpha_m\}$. Since each IS $\mathcal{I}_m$ associated with α_m corresponds to a solution of channel scheduling and radio resource allocation, solving $\mathcal{P}_\mathbf{1}$ leads to joint network throughput optimization and resource allocation. In addition, the solved variables together indicate the routing in network. The constraints and objective function are linear so that $\mathcal{P}_\mathbf{1}$ is a linear programming problem. From (1)-(8), one can see that the columns of the constraint matrix corresponding to $\{f_{(s,d)}(u,v)\}$ are fixed; while the columns associated with $\{\alpha_m\}$ are exactly determined by the ISs set $\mathcal{M}$.

However, in order to optimally solve the problem, we need to search for all the ISs to construct all the columns. The challenge is that the number of ISs can be extremely large as in the order of $O(2^{|\mathcal{T}|})$ (where $|\mathcal{T}|$ is the number of all tuples), and searching for all the ISs is an NP-hard problem [9]. Moreover, even if we can obtain all the ISs, the number of columns of the constraint matrix in the LP problem is so large that the generation and storage of the matrix is inefficient and even practically infeasible. A typical approach is to apply random search to find as many ISs as possible [1]. But this method can only give approximately optimal solutions and the computation overhead in finding reasonable number of ISs is very large. In this paper, rather than directly solving the LP problem after searching for a satisfactory amount of ISs, we explore the DCG method to quickly find solutions.

C. Energy-Efficient Capacity Optimization

The network capacity optimality may not necessarily lead to energy optimality since a high throughput often indicates high energy expenditure on data transmissions. Reducing energy consumption usually requires a decrease in the throughput. However, since different resource allocation solutions with different amounts of energy consumption may yield the same network capacity, it is possible to achieve energy conservation without degrading the network performance by carefully allocating resources. This leads to an interesting multi-objective optimization problem as the energy-efficient capacity optimization problem. In the following, we first present a model of the network energy consumption, and then establish the multi-objective optimization problem.

1) Energy model: In a time slot, suppose the nodes that all its incident links are unscheduled remain in sleep mode to save energy. The energy consumption of the network mainly consists of transmission energy and reception energy where we ignore the sleeping energy as well as the amount of energy spent on channel/radio/mode switching. For each link $l_{uv} \in \mathcal{L}$, denote P_u^t (P_v^r) as the amount of energy spent by the sender node u (receiver node v) for transmitting (receiving) one bit data. The energy consumption of a node depends on the amount of time that the node is involved in transmission which depends on the scheduling of ISs. Since we always schedule ISs, a tuple in some scheduled IS may not have data to transmit. Such tuples with zero flows can be switched off to further save energy. Notice that multiple tuples in the same IS and connected with the same node can be scheduled at the same time. In this case, the node may have to spend more energy to serve all the active tuples operating on different interfaces or channels. Taken these facts into consideration, the energy consumption of each node is actually proportional to the amount of incoming and outgoing flows at this node. Therefore the total amount of energy consumption E in a unit time period can be calculated as

$$E = \sum_{(s,d)\in\Omega} \sum_{l_{uv}\in\mathcal{L}} \left(P_u^t f_{(s,d)}(u,v) + P_v^r f_{(s,d)}(u,v) \right) \tag{10}$$

2) Multi-objective optimization formulation: Multi-objective optimization is conducted to deal with design problems which are characterized by the presence of multiple conflicting objectives. Unlike single-objective optimizations that search for global optimal solutions, multi-objective optimization instead seeks for a set of points that can fit the requirement of optimum. Such concept of optimum is known as the *Pareto optimality*, which is defined formally as follows [24]: *A point $\boldsymbol{x}^* \in \boldsymbol{X}$ is Pareto optimal if and only if there does not exist another point $\boldsymbol{x} \in \boldsymbol{X}$ such that $F(\boldsymbol{x}) \leq F(\boldsymbol{x}^*)$ and $F_i(\boldsymbol{x}) < F_i(\boldsymbol{x}^*)$ for at least one function*[3]. In other words, a solution is Pareto optimal if there is no other point that improves at least one objective function without detriment to another one. The set of Pareto optimal solutions forms the *Pareto front*. To solve the multi-objective optimization problem is to search for the Pareto optimal solutions or locate the Pareto front in the solution space.

We adopt the ϵ-constraint method to solve the multi-objective optimization where the problem is reformulated by keeping one of the objective functions and converting the

[3]If not specified, we are considering minimization problem.

others into constraints:

$$\begin{cases} \text{minimize :} & F_s(\mathbf{x}) \\ \text{subject to :} & F_i(\mathbf{x}) \leq \epsilon_i \quad \forall i = 1, \dots, k, i \neq s \end{cases} \tag{11}$$

where $F_i(\mathbf{x}), i = 1, \dots, k$, are the objective functions and $\{\epsilon_i\}$ indicate the constraints formed by the other objective functions. The solution to the revised problem can be proven to be Pareto optimal if the bounds of the converted constraints are all reached [21].

A systematic variation of ϵ yields a set of Pareto optimal solutions. We set energy consumption as objective function and convert capacity objective to constraint since the capacity of a network is naturally bounded by the network structure and configuration. In order to determine the value of ϵ, we need to first decide the upper bound of the objective function which in our case is the lower bound of the network capacity (since capacity is the objective to be maximized while the standard form is a minimization problem). It can be easily verified that in this model the solutions to the multi-optimization problem are Pareto optimal. Each solution on the Pareto front found in our problem stands for the optimal (minimal) energy consumption that can be achieved without degrading the current network capacity.

The value of ϵ cannot exceed the maximal capacity that can be achieved from the network. Therefore we can first solve the single objective optimization problem $\mathcal{P}_\mathbf{1}$ for optimal capacity λ^*, and then specify the value of ϵ by λ^* or some portion of λ^* such that $\epsilon = q\lambda^*$ and vary the value of q to get a variation of ϵ. Denote this multi-objective optimization problem as $\mathcal{P}_\mathbf{2}$:

$$\mathcal{P}_\mathbf{2} : \quad \begin{cases} \text{minimize :} & E \\ \text{subject to :} & \lambda \geq q\lambda^* \\ & \text{constraints (1)-(8) hold} \end{cases} \tag{12}$$

Let $\mathcal{M}^* \subset \mathcal{M}$, and $\mathcal{A} = \{\alpha_1, \dots, \alpha_{|\mathcal{M}^*|}\}$ be the scheduled set of ISs and their portions of time allocation in order to achieve the optimal capacity λ^*, respectively. Intuitively, if each IS $\mathcal{I}_m \in \mathcal{M}^*$ only uses $q\alpha_m$ portion of time to transmit data, the amount of flow of each scheduled tuple will decrease by a percentage of $1 - q$. Thus, it is easy to see that, in this case, the constraint (11) holds while the energy consumption decreases to qE based on (10). Therefore, the term q also indicates an upper bound for the solution of optimal energy consumption to problem $\mathcal{P}_2$.

As an alternative to $\mathcal{P}_2$, if each node has a pre-defined energy budget, we can convert energy objective to constraint according to the network requirement and treat capacity as objective function. Such conversion will yield a similar problem which can be solved in a similar way as below.

V. Energy-Efficient Capacity Optimization Algorithms

Above we have formulated the MCF problem as linear programming problems. Despite the extremely large number of all ISs, our experiences indicate that some critical ISs will be actually scheduled while most ISs may never be used. This inspires us to adopt the efficient DCG method [11] to iteratively generate columns that are expected to improve the problem objective. The generated columns together with an initial set of feasible columns form a subspace where the optimal solution can gradually approach to the optimal solution in the whole problem space.

A. Delayed Column Generation

The DCG implementation involves a sequence of master problems and sub-problems. We describe the DCG method in the following LP framework (the master problem) with the understanding that problem $\mathcal{P}_2$ is a special case.

$$\begin{aligned} \text{minimize :} \quad & \sum_{i \in Q} c_i x_i \\ \text{subject to :} \quad & \sum_{i \in Q} a_{ki} x_i \leq b_k, \quad k = 1, \dots, K \\ & x_i \geq 0, \quad \forall i \in Q \end{aligned}$$

where Q is the index set corresponding the columns that have already been generated and $\{x_i\}$ is the set of variables to be determined with the corresponding costs $\{c_i\}$. Also, suppose there are K constraints (K rows in the coefficient matrix) with bounds $\{b_k\}$. a_{ki} is the (k,i)-th entry of the constraint matrix. The master problem is a relaxed version of the original problem with a subset of columns.

The DCG method starts with an initial basic feasible solution of the original problem and an initial set of columns corresponding to Q associated with the solution. Then, the method runs iteratively. In each iteration, after solving the master problem, it searches among the columns outside Q for a new column with negative reduced cost $\bar{c}_s$ to enter Q, where the reduced cost of a column (with index s) is expressed as

$$\bar{c}_s = c_s - \sum_{k=1}^{K} \beta_k a_{ks}, \tag{13}$$

and $\{\beta_k\}$ are the optimal dual variables to the master problem. If no such column is found, the method exits with the current $\{x_i\}$ as the optimal solution. Otherwise, since there may be many such columns, the one with the most negative reduced cost is selected by solving the following sub-problem:

$$s^* = \operatorname*{argmin}_{s \notin Q} \bar{c}_s = \operatorname*{argmin}_{s \notin Q} (c_s - \sum_{k=1}^{K} \beta_k a_{ks}). \tag{14}$$

After solving the sub-problem, the master problem is updated by adding s^* to Q and generating the corresponding column. Then a new iteration is triggered.

B. The Proposed Algorithms

Based on the DCG method, we propose two algorithms for the energy-efficient capacity optimization, namely DCG-S and DCG-W algorithms. While both the two algorithms solve problem $\mathcal{P}_\mathbf{1}$ at first to obtain the optimal capacity λ^* by taking advantages of DCG for its efficiency, they are different mainly in treating problem $\mathcal{P}_2$.

1) Solving $\mathcal{P}_{\mathbf{1}}$: In both DCG-S and DCG-W algorithms, $\mathcal{P}_{\mathbf{1}}$ is solved with the same DCG-based method. The DCG iterations should start with an initial basic feasible solution. First, the initial set of ISs can be constructed by repeating the following procedure until all the tuples are covered by the ISs found: randomly select a tuple that has not been included in any of the existing ISs and then search an IS starting from this tuple. Pseudo codes for this method is shown in Algorithm 1.

Algorithm 1: Finding an initial set of MISs.

```
T: set of all tuples;
S = ∅. //solution: initial set of ISs;
V = ∅. //set of tuples contained in selected ISs;
while T/V ≠ ∅ do
    p ← a tuple randomly chosen from T/V;
    I = {p}; V = V ∪ {p} ;
    while p' ∈ T/V do
        if {p, p'} is an IS then
            I = {I, p'};
        end
    end
    S ← {S, I};
end
```

Suppose n ISs have been selected above, we can construct the initial feasible solution that schedule them alternately in the following conservative manner. Assign equally $\frac{1}{n}$ portion of time to each IS such that each tuple in the selected ISs will gain $\frac{1}{n}$ physical capacity on that tuple. Then, to ensure feasibility, each commodity flow $f_{(s,d)}(u, v)$ on link l_{uv} can be set as the bottleneck capacity of a path between the corresponding source and destination pair (s, d). With the above scheduling and flow assignment, we can therefore construct a feasible solution as the initial solution, and the problem $\mathcal{P}_{\mathbf{1}}$ can be then solved following the standard DCG iterations as described above.

The constraint matrix corresponding to this solution consists of all the columns associated with capacity constraints, flow conservation constraints and the initial set of ISs. Any newly generated column is constructed by an IS, so that generating a new column is in fact finding a new IS. Recall the objective of $\mathcal{P}_{\mathbf{1}}$. The cost coefficient c for the variable α_m that corresponds to an IS is 0. Therefore, finding a new IS by solving the sub-problem in (14) reduces to solving the following maximum weighted independent set (MWIS) problem:

$$\max_{s \notin Q} \sum_{k=1}^{K} \beta_k a_{ks}. \tag{15}$$

To solve the above MWIS problem, we adopt the greedy algorithm [25] which runs iteratively as follows. In each iteration, it collects the tuple with minimum weight degree (sum of weights of all its neighbors) and then removes all the tuples with edges connected to the collected tuple from the conflict graph. It returns an MWIS when all the tuples have been collected or removed. Algorithm 2 gives a formal description of this method.

Algorithm 2: Greedy MWIS algorithm

```
G_c: the conflict graph;
S = ∅
while G_c ≠ ∅ do
    Let p be a tuple of minimum weighted degree in G_c;
    S = S ∪ {p};
    Remove p and its neighbors from G_c;
end
Output: S;
```

2) DCG-S algorithm: The solution of $\mathcal{P}_{\mathbf{1}}$ provides us the optimal capacity λ^* and a set of ISs that are selected to obtain λ^*. With the DCG process, these selected ISs construct a subspace $\mathcal{M}_{sub}$ of the original $\mathcal{M}$. Our formulation of the multi-objective optimization problem gives the intuition that we should schedule as less tuples as possible in order to minimize energy consumption. It is hence a simple way to restrict our attention to the subspace $\mathcal{M}_{sub}$ in solving $\mathcal{P}_{\mathbf{2}}$ because the optimal capacity has already been achieved by $\mathcal{M}_{sub}$. In the DCG-S Algorithm, with $\mathcal{M}_{sub}$ obtained above, $\mathcal{P}_{\mathbf{2}}$ is solved simply by applying the LP method. Usually, the number of ISs in $\mathcal{M}_{sub}$ is small, so the complexity of the DCG-S algorithm is almost in the same order as that of the DCG-based algorithm in solving $\mathcal{P}_{\mathbf{1}}$.

3) DCG-W algorithm: The lightweight DCG-S algorithm based on our intuition cannot guarantee to find the global optimal solution to the multi-objective problem, because the energy optimization in $\mathcal{P}_{\mathbf{2}}$ is solved based on the capacity-oriented subspace $\mathcal{M}_{sub}$. For global optimality, it is necessary to re-select ISs with respect to energy optimization. Therefore we propose the DCG-W Algorithm indicating that $\mathcal{P}_{\mathbf{2}}$ is solved based on the whole problem space $\mathcal{M}$. In DCG-W, we first solve $\mathcal{P}_{\mathbf{1}}$ as above but only store the obtained value of λ^* which is used to construct $\mathcal{P}_{\mathbf{2}}$. We independently perform another series of DCG iterations to solve $\mathcal{P}_{\mathbf{2}}$, where the problem space is re-initialized and the columns are re-defined according to the parameters in $\mathcal{P}_{\mathbf{2}}$. The initial feasible solution can be the solution achieved in the first DCG iterations for solving $\mathcal{P}_{\mathbf{1}}$. Compared with the DCG-S algorithm, one can see that the DCG-W algorithm incurs higher computation complexity but better energy optimality.

VI. Numerical Results

In this section, we present numerical results to demonstrate the performance of our optimization methods. We consider an MR-MC network with 25 homogeneous nodes randomly deployed in a $1000m \times 1000m$ area as shown in Fig. 1. Each node by default is equipped with 3 radio interfaces that can operate on 8 wireless channels; however, in the following, we vary these numbers to evaluate their impacts on the performances. The physical link capacity on each channel is set to 1 rate unit. The communication range and interference range of each node are $250m$ and $500m$, respectively. Consider that there are 3 commodity flows with the same flow requirement (3 rate unit) traversing through the network, where the source-destination pairs are denoted as $(S_1, D_1), (S_2, D_2)$ and (S_3, D_3) as shown in Fig. 1. The unit energy consumptions for transmitting and receiving a unit amount of data are set as $P^t + P^r = 1$.

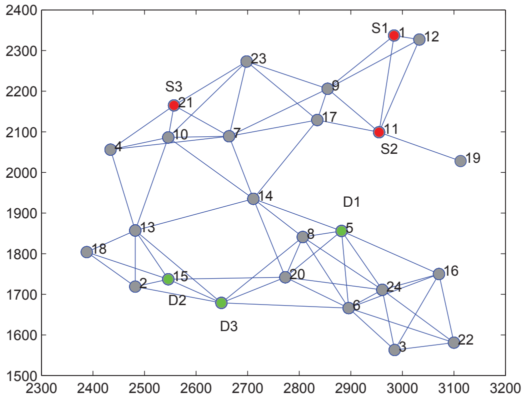

Fig. 1. Network topology.

We compare the proposed DCG-S and DCG-W algorithms with the RS algorithm [16]. In the RS algorithm, we randomly select 2×10^5 distinct ISs. These ISs are then expanded to maximum independent sets (MISs) based on which the problem $\mathcal{P}_1$ is solved. The reason is that each expansion of an IS will introduce more mutually independent tuples for data transmission that do not conflict with any existing tuples in the IS, and hence may improve (or at least maintain) the achieved capacity. In the DCG-based algorithms, we do not conduct such expansion since each IS is optimally determined by the sub-problem solutions in the DCG algorithm itself. The DCG iterations involved in our two algorithm terminate when no new column can be found by the MWIS algorithm (Algorithm 2) or the corresponding objective value holds for 100 iterations. We develop Matlab programs to implement the above algorithms where the CPLEX [26] tool is embedded within the Matlab environment for solving the LP problems involved in the algorithms.

A. Energy Efficiency Gain

In order to demonstrate the energy efficiency improvement of multi-objective optimization, we first perform single-objective capacity optimization (problem $\mathcal{P}_1$) with RS method. The value of energy efficiency achieved in this method (raw energy efficiency) is used as a reference value for later comparisons. The multi-objective optimization (problem $\mathcal{P}_2$) is performed with the value of q in constraint (12) set as 100%. The achieved energy efficiency is compared with the reference value to obtain the energy efficiency gain. The computation runs independently in different scenarios with various numbers of radios on each node or numbers of channels in the network.

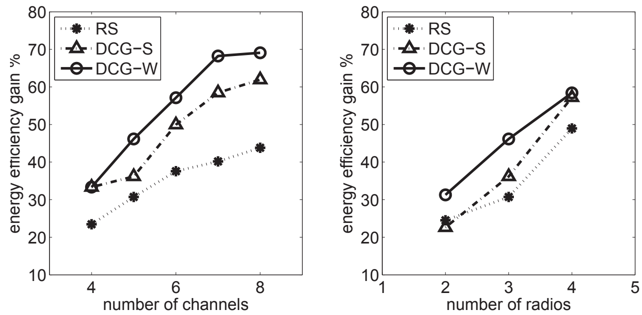

Fig. 2. Energy efficiency gain.

The results of energy efficiency gain are shown in Fig. 2, from which we observe that by solving the multi-objective optimization we can always achieve at least 20% improvement of energy efficiency comparing with the energy-blind single-objective optimization. Due to the complex structure and interference relationship of multi-hop networks, there may be multiple resource allocation solutions achieving the same network capacity but consuming different amounts of energy. The energy-blind capacity optimization will return any one of the solutions while the multi-objective optimization can obtain the one with minimal energy consumption and therefore achieve higher energy efficiency. As the number of radios or channels increases, the network structure becomes more complicated and there is a higher probability that the capacity-only optimization finds a much worse solution than the multi-objective optimization. As a result, the energy efficiency gain is much larger when there are more radios or channels available and there can be 60% or even more improvement in some cases.

Above we can also observe that the DCG-W algorithm achieves the most energy efficiency gain while the RS algorithm achieves the least. Performance comparisons among these algorithms are examined in more details in the following.

B. Performance Comparison

To compare the performance achieved by the RS and DCG-based algorithms, we first investigate the achieved optimality in terms of network capacity (by solving $\mathcal{P}_1$). Since DCG-S and DCG-W methods share the same procedures in this stage, here we term them as DCG-based algorithm. The results are shown below in Fig. 3, where, on each line, each point stands for the achieved energy and capacity at a number of available channels (4 to 8 for the points from the left to the right). It can be seen that, with the same number of channels, the DCG-based method can always achieve higher capacity than the RS algorithm. Meanwhile, the solution of DCG-based method consumes even less energy than that of RS algorithm. This is because the former can fully explore the network to figure out the best resource allocation and scheduling while the latter is limited to a subspace (even though we have searched a significant number of ISs, the whole space is still much larger that an exhaustive search is almost infeasible). Such advantages of the DCG-based method over the RS algorithm becomes even more significant as the number of channels increases, as shown in the figure that the performance gaps in terms of both capacity and energy are being widened.

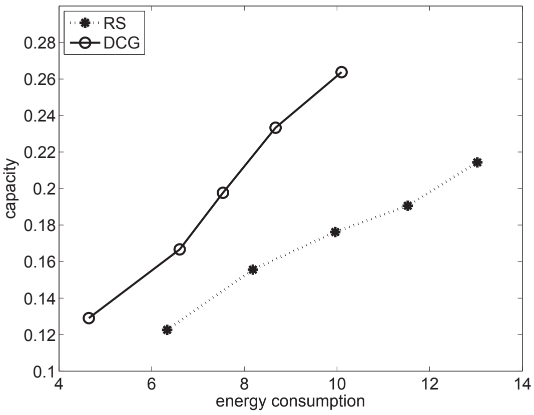

Fig. 3. Capacity and energy comparison.

We then compare the performances in solving the multi-objective problem $\mathcal{P}_2$ and derive the Pareto fronts solved by each method. From the results shown in Fig. 4, we can observe that the Pareto fronts solved by the DCG-S and DCG-W algorithms are much closer to the upper left corner than those by the RS algorithm, where the closer to the upper left corner the higher capacity and lower energy consumption are achieved.

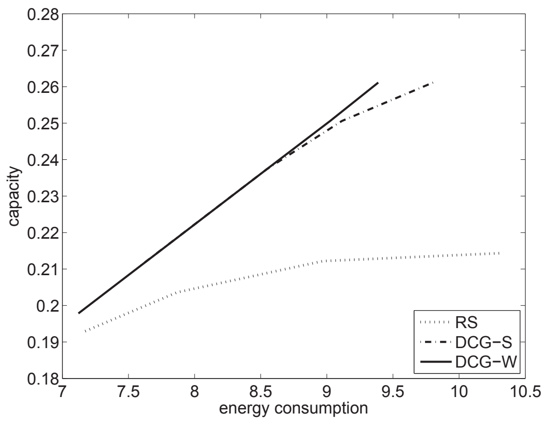

Fig. 4. Pareto front comparison.

In the following, we closely examine these algorithms by using the *normalized energy efficiency* metric, where the normalization is done by dividing the achieved energy efficiency by the corresponding reference value (or raw energy efficiency). The comparison is performed in four groups of parameter settings with different number of nodes, channels or value of q. From the results shown in Fig. 5, we have the following observations of these algorithms:

1) The RS algorithm is outperformed by the proposed DCG-based algorithms in most cases since the random searching only uses the information of a subspace of the entire IS space. Especially when the network scale increases, the portion of the whole IS space covered by the searched subspace becomes even smaller, resulting that the solutions obtained get further away from the optimum.
2) The DCG-S algorithm can achieve better performance than RS in most cases since global IS information is considered in solving $\mathcal{P}_1$ for optimal capacity. However, it still suffers from the similar issue as in the RS algorithm that only a subspace generated in the first step is used in the LP step for energy optimization.
3) The DCG-W algorithm achieves the best performance in all cases. In this algorithm, the column generation iterations are performed twice where both capacity and energy are involved in the optimizations. Compared with DCG-S, the DCG-W algorithm fully explores the problem space both in capacity optimization and energy optimization steps.

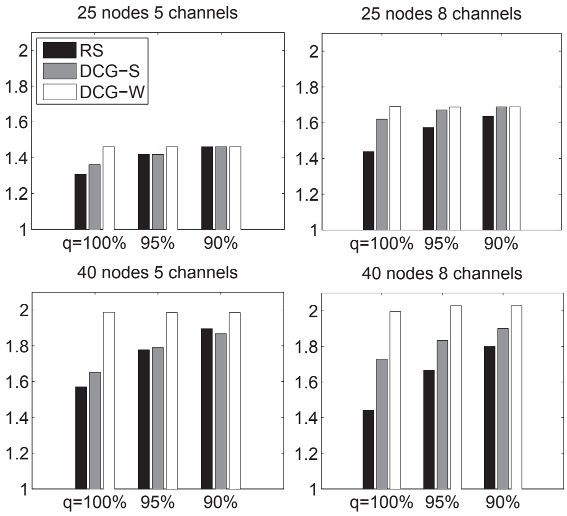

Fig. 5. Normalized energy efficiency achieved by the algorithms.

Furthermore, Fig. 5 also demonstrates the impact of q on the performances obtained by these algorithms. We can see that, in general, the energy efficiency achieved by the RS and DCG-S algorithms tends to increase as q decreases. As q gets lower, the capacity constraint is relaxed which in turn results in that more choices of resource allocations can be considered for energy optimization. Consequently, it is easier to get the most energy efficient configuration of network flow and scheduling which leads to higher energy efficiency. In contrast, the energy efficiency of the DCG-W algorithm is almost not affected by the changing of q, since this method can always maintain optimality.

C. Computation Time Comparison

Table I summarizes the comparisons of these algorithms' computation time. In the first network, the computation time

TABLE I
COMPUTATION TIME COMPARISON

network with 25 nodes (each has 3 radios) and 5 channels	
method	computation time (seconds)
RS	1015
DCG-S	90
DCG-W	196
network with 40 nodes (each has 3 radios) and 8 channels	
method	computation time (seconds)
RS	30193
DCG-S	970
DCG-W	1156

of the RS algorithm is about 5 to 10 times of that in DCG-based algorithms; while in the second network with larger scale, the computation time consumed by the RS algorithm climbs about 30 times while the other two algorithms increase by 5 to 10 times. The computation time of the RS method is mainly consumed by the searching of ISs and the large-scale LP optimization performed based on these ISs. However, the DCG-based algorithms only select a moderate number of ISs in the first stage. Moreover, the computation time of the DCG iterations is low because the greedy algorithm that solves the MWIS problem is expressively fast. Also, in the second stage that solves $\mathcal{P}_2$, the optimization processes of the proposed algorithms start with a small scale problem with much less columns than that in the RS algorithm. As a result, the proposed algorithms can save much computation time in both of the two stages.

VII. CONCLUSION

In this paper, we investigate the energy-efficient capacity optimization problem in generic MR-MC wireless networks, and formulate a multi-objective optimization framework based on the MCF problem scenario. Our framework is augmented with the tool of MDCG that facilitates joint design of routing, channel/radio scheduling and flow allocation. A significant challenge regards to the large problem scale due to that there could be extremely large number of ISs. To tackle this, we propose two DCG-based algorithms that can efficiently solve the multi-objective optimization problem without searching for too many ISs. Extensive numerical results demonstrate that the proposed algorithms achieve promising energy efficiency gain. Moreover, the proposed DCG-W algorithm achieves the best energy efficiency over the RS and DCG-S algorithms.

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