Abstract—This paper details the progressive development of symbol shaping for Barker spread IEEE 802.11 modulation used in wireless fidelity communications. Symbol shaping is used to satisfy the spectral mask requirements of the Federal Communications Commission (FCC) with minimal output filtering and inter-symbol interference. Logarithmic, sinusoidal, and sinc-function shaping is investigated using analytic, simulation, and experimental methods. Power spectral densities are compared to the FCC mask to determine the effectiveness of the symbol shaping. Bit error rate is evaluated to provide a performance metric for each symbol shape. A complete experimental system has been implemented as a test bed for this research.

I. INTRODUCTION

Digital wireless communication networks have proliferated exponentially in the last few years. The unlicensed 2.4 GHz Industrial, Scientific and Medical (ISM) band [1] is dominated by high-speed digital communications systems that adhere to the IEEE 802.11 standards [2]. These standards were implemented to provide reliable, mobile, and secure wireless connectivity at desirable data rates. Wireless Local Area Networks (WLANs) are now found in a myriad of environments – anywhere from office buildings to schools and residential complexes, and even coffee shops. In addition to WLANs, a host of other wireless devices transmit in the ISM bands, for example, cordless telephones, Bluetooth [3] headsets, wireless video game controllers, baby monitors, intercom devices, and many wireless security cameras.

Since many wireless devices operate in the same ISM band, overcrowding is becoming a more pervasive problem, leading to interference that degrades the performance of IEEE 802.11 systems. This problem is compounded by the presence of several unintentional, non-data carrying interferers (such as microwave ovens [4]) that radiate in the 2.4 GHz band. In an attempt to limit the interference issues, the Federal Communications Commission (FCC) has implemented strict guidelines that constrain the power and bandwidth of all devices using the ISM band.

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II. BARKER SYMBOL SHAPING

The Barker spreading code used in IEEE 802.11 systems provides a processing gain of 11, and thus it expands the signal bandwidth by a factor of 11. Each data bit at the 1 Mbps rate is transmitted as a sequence of 11 Barker chips with the chip interval, $T_c = T / 11$, where $T$ is the bit duration (1 µs). The main-lobe baseband signal bandwidth for 1 Mbps data is 11 MHz.

The Barker chip sequence used in the 802.11 standard is:

$$B = [+1, -1, +1, +1, -1, +1, +1, -1, +1, -1, -1]$$

Several smoothing functions were applied to the Barker waveform. A logarithmic smoothing function was used for the rising and falling transitions. The general form of this function was

$$p_L(t) = k_0 + k_1 \log(at + t_0)$$

where $k_0$, $k_1$, $a$, and $t_0$ are constants.

Plotted in Fig. 2 is the logarithmically shaped Barker symbol and the (original) rectangularly shaped Barker waveform. The amplitude of the log-shaped waveform has been adjusted so that the energy per symbol is the same in both cases.

Sinusoidal pulse shaping was also employed. The resulting waveshape is shown in Fig. 3, where again the energy per symbol is equal to that of the original Barker waveshape. The sinusoidal pulse shape can be expressed analytically as

$$p_S(t) = 1.414 \sum_{n=1}^{4} p_{S_n}(t)$$

where the compound parts are given as

$$
\begin{align*}
    p_{S_1}(t) &= -\sin(2\pi t / 2T_c), & -5T_c \leq t \leq -3T_c \\
    p_{S_2}(t) &= -\cos(2\pi t / 4T_c), & -3T_c \leq t \leq -T_c \\
    p_{S_3}(t) &= \sin(2\pi t / 2T_c), & -T_c \leq t \leq 0 \\
    p_{S_4}(t) &= \sin(2\pi t / 6T_c), & 0 \leq t \leq 6T_c
\end{align*}
$$

In most Wireless Fidelity (Wi-Fi) systems, the chips are unit amplitude rectangular pulses, and the PSD for the unfiltered data signal modulated by this Barker waveshape does not satisfy the FCC spectral mask. Figure 1 shows that the second lobe must be filtered by at least 17 dB and the third lobe by at least 32 dB to be below the mask. In our simulation studies, a fifth order Butterworth filter with a cutoff frequency of 9.5 MHz was required to satisfy the mask requirements. This introduced considerable ISI.

The Barker waveform was modified by smoothing the rectangular pulse shapes in the original Barker symbol. The modified pulse shapes still adhered to the general Barker sequence and maintained good autocorrelation properties, as shall be discussed in Section IV.
A third shaping was studied, with sinc-functions, of the form
\[ p_s(t) = A \left[ \text{sinc}(bt + b_s) \right]^m, \]  
(4)
where \( 0 < m < 1 \). The best PSD, relative to the FCC mask, was obtained with two values for \( m \), where \( m_1 = 0.55 \) for the 1-chip segments, and \( m_2 = 0.83 \) for the 2- and 3-chip segments, e.g., +1 +1 +1.

For each pulse shape, the PSD was analyzed via simulation and experimentation to identify the pulse shape that provided maximum sideband attenuation. Regardless of the exact form of the Barker symbol shape, the baseband data signal can be represented as:
\[ y(t) = \sum_{n=-\infty}^{\infty} x(n) \cdot p(t - nT) \]  
(5)
where \( x(n) \in \{-1, 1\} \) is random binary data, that is independent and identically distributed, and \( p(t) \) is the pulse shaped Barker waveform.

The signal \( y(t) \) is a pulse amplitude modulated signal with amplitudes +1 and −1 and is a zero-mean cyclostationary random process. Consequently, its PSD is given as [8]:
\[ S_y(f) = \frac{\sigma_x^2}{T} |P(f)|^2, \]  
(6)
where \( \sigma_x^2 \) is the variance of \( x(n) \) and \( P(f) \) is the Fourier transform of the pulse shaped Barker symbol, \( p(t) \).

A closed form analytic expression cannot be obtained for the PSD of the logarithmically shaped Barker symbol. Its PSD was found with a MATLAB® simulation using the Welch PSD [8]. This result is plotted in Fig. 4 using 5000 data bits. This PSD shows an improvement in spectral characteristics, where the sidebands are attenuated 8 dB more than the rectangular Barker PSD shown in Fig. 1. A third order Butterworth filter (with a 9.5 MHz cutoff frequency) is needed to satisfy the spectral mask.

The sinusoidally shaped Barker waveform lends itself to an analytic study. Using rectangularly windowed and shifted sinusoids, as defined in (3), the Fourier spectrum of \( p_s(t) \) is found to be
\[ P_s(f) = \pi T \sum_{k=1}^{2} \left[ \left( \frac{k}{2} \cos(k\pi f T_s) e^{j\pi f T_s} \right) \left( k\pi f T_s - \left( \frac{\pi}{2} \right)^2 \right) \right. 
\] 
\[ \left. - j \left( 4k-2 \right) \sin(4k-2)f T_s e^{-j(4k-2)f T_s} \right] \]  
(7)
Using (6) and (7) the PSD can be computed. The analytic PSD is displayed in Fig. 5, and the MATLAB simulated PSD (with 5000 bits) is plotted in Fig. 6. The agreement is excellent and we observe that there is an 11 dB attenuation improvement over the rectangular Barker PSD. To satisfy the FCC mask requirement a simple second order Butterworth filter with a 9.5 MHz cutoff frequency can be used.

![Fig. 4. Simulated PSD of the logarithmic Barker waveform.](image1)

![Fig. 5. Analytic PSD of sinusoidal pulse shaped Barker waveform.](image2)

![Fig. 6. Simulated PSD of sinusoidal pulse shaped Barker waveform.](image3)
As with the log-shaped Barker symbol, the sinc-function shaped symbol does not lend itself to a closed form analytic expression for the PSD. Simulation was used to find the optimal $m_1$ and $m_2$ values for the best spectral characteristics. The simulated PSD, with 5000 data bits, is plotted in Fig. 7. Observe that the first sideband is attenuated by 12 dB compared to the rectangular Barker pulse, but there is also less influence on the third sideband. A second order Butterworth filter with 9.5 MHz cutoff frequency was adequate to meet the spectral mask in this case.

![Baseband Transmitted signal PSD](image)

**Fig. 7.** Simulated PSD of sinc-function shaped Barker waveform.

### III. EXPERIMENTAL TEST BED

The simulations performed in Section II were all done using MATLAB software for the four Barker waveshapes, i.e., rectangular, logarithmic, sinusoidal, and sinc-function. In all cases, the modulated signal power was referenced to 1 watt, and a correlator detector was employed after synchronous demodulation. An additive white Gaussian noise channel was also simulated to allow for BER evaluation, included in Section IV. This section presents the experimental test bed used to confirm our simulation results.

A complete emulation system was constructed in the Wireless Network and Communication (WiNCom) Research Center at Illinois Institute of Technology (IIT) using Comblock modules [9]. Comblocks are communication chipsets that can be connected as blocks to construct transmitters and receivers. The transmitter, shown in Fig. 8, consists of five chipsets connected in series: a computer interface module, an arbitrary waveform generator, a high-speed baseband Digital-to-Analog Converter (DAC), a mixer operating in the 2.4 GHz band, and an amplifier. The receiver, displayed in Fig. 9, consists of three chipsets. At the RF end, one chipset digitizes the 2.4 GHz signal received and stores it in the next chip, which is finally downloaded to a Personal Computer (PC) via a computer interface chipset. Dipole antennas connected to the Comblock transmitter and receiver allow transmission in the 2.4 GHz ISM Band.

Due to hardware speed limitations, a 4 MHz chip-rate was used corresponding to a bit-rate of 363 kbps with the 11 chip Barker code. To experimentally verify the simulation results, a Rohde & Schwarz spectrum analyzer (model no. FSP 38) was used to obtain the experimental PSD. Using 4 Mchips/second, the main-lobe bandwidth in the experimental PSD is expected to be 4 MHz. The baseband modulated signal was viewed with a 400 MHz oscilloscope to provide a temporal domain representation. At the receiver, the demodulated, digitized waveform was analyzed and subjected to a correlator detector via a MATLAB® program on the PC. This permitted an experimental evaluation of the BER. The received signal’s strength could be varied by changing the transmitter-to-receiver distance, effectively changing the Signal-to-Noise Ratio (SNR) of the received signal. The BER results are presented in Section IV.

![The Comblock transmitter used for experimental emulation.](image)

**Fig. 8.** The Comblock transmitter used for experimental emulation.

![The Comblock receiver.](image)

**Fig. 9.** The Comblock receiver.
An experimentally measured PSD using a sinusoidally shaped Barker waveform, modulated at 2.420 GHz, is given in Fig. 10, along with the FCC spectral mask (dashed lines). The analytic and simulated PSD, in Figs. 5 and 6, respectively, match very closely with the experimental PSD. Note that the experimental emulation PSD in Fig. 10 has a mask with transitions at 4 MHz and 8 MHz away from the carrier frequency since the chip rate is 4 MHz.

This experimental work has shown that the implementation of complex pulse shaping in a transceiver system is feasible. Such system hardware can be constructed inexpensively by replacing the high-speed DAC with a discrete-time analog storage device. Such a device stores the modulator’s signal level values as analog voltages. During each bit interval, the analog voltages will be output at discrete sub-time intervals to construct the complete pulse shape.

Typically, a communication system requires a small finite set of pulse shapes. Thus only a limited number of the analog storage cells are needed, thereby eliminating the need for complex digital logic circuits and DACs. Recently a topic of promising research has been the integration of such analog waveform generators with digital communication systems [10].

IV. BER PERFORMANCE

To illustrate the detection process for Barker spread binary data, the autocorrelation function was determined for the different shaping functions. With equal energy per shaped symbol, the peak of the autocorrelation function should be the same. Plotted in Fig. 11 is the autocorrelation function of the rectangular Barker waveform (dashed line), and the sinusoidally shaped Barker waveform (solid line). The Barker code’s autocorrelation property dictates that the autocorrelation function is bounded by one-eleventh of its peak for time shifts of 1 chip or more. This property is largely preserved with the sinusoidally shaped Barker waveform, and is strictly preserved for time shifts of 3 chips or more. Consequently, the shaped Barker communication system should be robust to multi-path distortion and noise.

BER simulation studies were performed for each of the four Barker shaped waveforms in MATLAB® at various SNR levels. A synchronous demodulator and correlator detector were used in all cases. The results are shown in Table 1 using 50,000 random test bits each time. This table also indicates the filter order used to achieve the FCC spectral mask requirement after pulse shaping. With low order filters, ISI is minimized; but ISI will increase as the filter order grows. High noise levels were chosen to obtain meaningful BER results with the available computing resources. The experimental BER results, obtained using the Comblock test bed, are shown in Table 2.
Table 2. Experimental BER measurements at receiver-to-transmitter distance of 1 meter.

<table>
<thead>
<tr>
<th>Pulse Shape Used</th>
<th>Experimental BER</th>
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</thead>
<tbody>
<tr>
<td>Rectangular</td>
<td>9.99E-03</td>
</tr>
<tr>
<td>Logarithmic</td>
<td>6.22E-03</td>
</tr>
<tr>
<td>Sinusoidal</td>
<td>3.71E-03</td>
</tr>
<tr>
<td>Sinc-function</td>
<td>5.84E-03</td>
</tr>
</tbody>
</table>

In general, the sinusoidally shaped system performed the best. This system experienced little ISI due to the low order filter used. The rectangular Barker system required the highest order filter and thus experienced more ISI that degraded the performance. The value of symbol shaping versus output filtering to satisfy the FCC spectral mask has thus been firmly established.

V. LINE CODING & PULSE SHAPING

Although the sinusoidal Barker waveform shaping was able to reduce the PSD considerably over the rectangular shaped Barker symbol, to achieve FCC mask compliance a second order output filter was still needed. The dominant feature of the PSD of the original Barker waveform comes from the abrupt transitions after just one chip interval, that is after 1 μs. This feature is also seen with the shaped Barker symbols, and is illustrated in the experimentally recorded oscilloscope plot in Fig. 12, where sinusoidal shaping has been used. The sudden discontinuity in the time domain raises the power of the higher frequency components in the spectral domain. Thus, to eliminate the need for an output filter to satisfy the FCC spectral mask, it is necessary to eliminate these discontinuities.

VI. CONCLUSIONS

A new Barker spread modulation scheme was investigated that incorporated pulse shaping techniques with an 11-chip Barker code. Four pulse shapes were studied and their PSDs determined. In all cases the PSD was compared to the FCC spectral mask. The sinusoidally shaped Barker waveform required the least output filtering in order to satisfy the spectral mask. The BER performance was also studied. The pulse shaped systems performed better in several respects: better spectral characteristics, lower order filter requirement, and improved BER performance. Again, the best performance was obtained with sinusoidal pulse shaping. The conclusions were verified by matching results obtained by analytic, simulation, and experimental studies.

REFERENCES